Six-Phase Synchronous Machine Model in Machine Variables and "d-q" Variables

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1 Motivation/Introduction

This brief paper seeks to discuss three questions regarding six phase synchronous machines.

First, what are the advantages, if any, of using a d-q analysis in the simulation of transient behaviour of a six phase machine? If a waveform approach is being used in the simulation, is the d-q analysis necessary? Will the d-q analysis simplify or quicken the required computations? Will the d-q analysis improve the accuracy or rate of convergence of the solution?

Second, in the way of the first questions, what are the underlying principles and assumptions of the d-q analysis? Would these underpinnings limit the utility of the d-q analysis in simulating large (distant from steady state equilibria) transient effects?

Lastly, what are the peculiarities of six phase machine applications? How is the six phase machine different from the more familiar three phase machine? How is it similar? What do the differences and similarities portend?

The second section of this paper provides a six phase synchronous machine model. The components of this model, with their appropriate assumptions, are described. The third section of this paper develops the transformation to rotor coordinates of the flux linkages, voltage equations and torque relationships. The final section of the paper seeks to determine which of the analyses are best suited for simulation.

2 Six Phase Synchronous Machines

2.1 Why Multiphases?

A single phase stator winding, excited by a sinusoidal current source, produces forward and backward travelling MMF waves in an air-gap. (A synchronous **motor** mind-set is employed here.) The rotor field can seek to align with either the forward or backward travelling MMF wave. Adding an additional identical phase stator winding in space quadrature and exciting it with a current in time quadrature of equal magnitude produces a sole forward travelling MMF wave in the air-gap.--This fulfills the requirement for average power conversion. Having additional phases beyond two in quadrature is a generalisation of the rotating MMF wave derivation.

Three phase power, or any $3 \cdot n \quad n = 1, 2, 3, \ldots$, is economical to generate and transmit across distances encountered in utility applications. For this and other reasons, three phase power is somewhat pervasive. Three phase synchronous machines too are pervasive. Hence, the tools for analysis of three phase machines are well developed.

Six phase power is easily obtained from three phase power. In high power applications, increasing the number of phases can ease the burden of current density limitations on conductors of a given size. Hence, an application of six phase machines is a high-power ship propulsion system. The analysis tools for three phase machines provide a basis for the analysis of six phase machines.

2.2 Six Phase Synchronous Machine Model

2.2.1 Types of Six Phase Synchronous Machines

Merely to say "a six phase synchronous machine" does not provide information enough to specify a machine. There exist symmetrical and asymmetrical six phase machines. The difference between these two machines is shown in Figure 1. Both will be considered in this treatise.

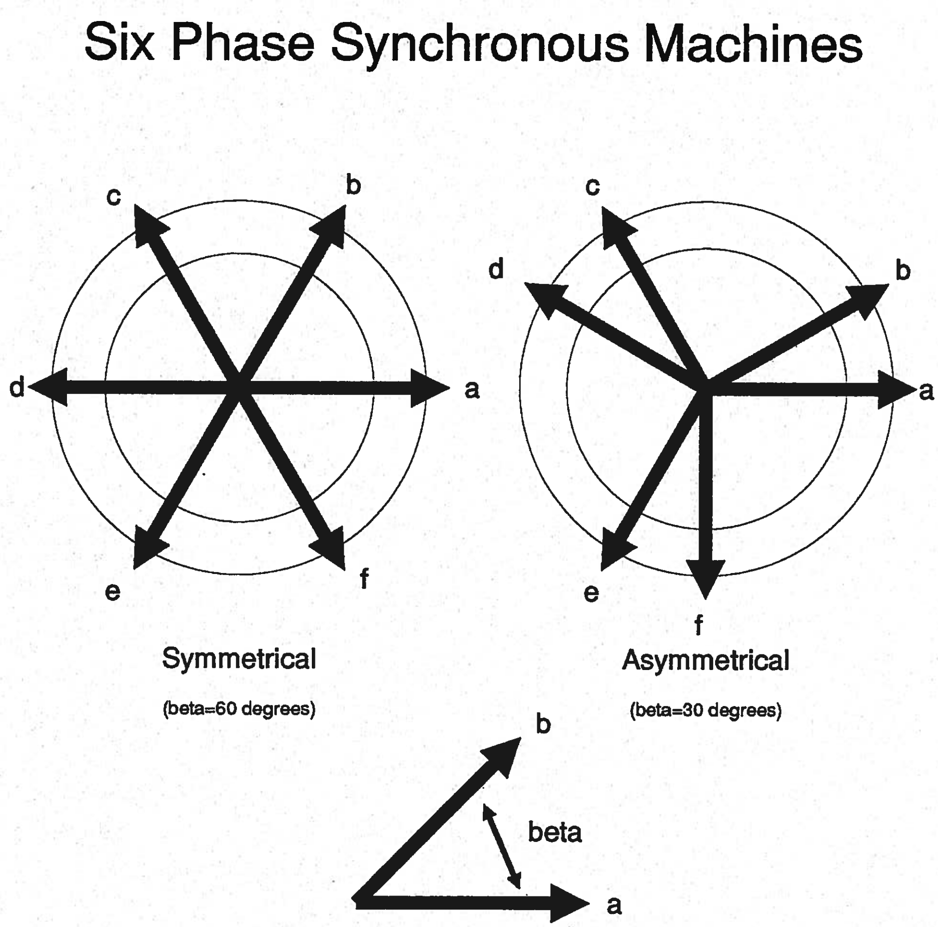

Figure 1

The angular displacement β in the two types of machine plays an important role in the stator winding mutual inductances. It greatly complicates the mathematics as well.

Given a symmetrical or an asymmetrical six phase synchronous machine, the machine can be a round rotor or a salient pole machine. This difference will have a large impact on the form of the inductance matrices. Both types will be considered in this treatise.

2.2.2 Flux/Current Relationships

To determine the forces of electromagnetic origin using the classic coenergy approach, the magnetic flux within the air-gap of the machine must be described as a function of the phase currents and the rotor angle. A magnetically linear machine is assumed outright. Further, only the fundamental space and time components of the Fourier series expansion of the air-gap fluxes will be considered. Hence, this model would not suffice for an application which requires information about harmonics. In light of these assumptions, the fluxes within the six phase machine can be described as below.

$$\begin{bmatrix} \underline{\lambda}_{ph} \\ \underline{\lambda}_{R} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{L}}_{ph} & \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^T & \underline{\mathbf{L}}_{R} \end{bmatrix} \begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_{R} \end{bmatrix} \qquad \#1$$

The phase quantities carry a "ph" subscript and the rotor, or field, quantities carry a "R" subscript.

$$\underline{u}_{ph} = \begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \end{bmatrix} \qquad \underline{u}_R = \begin{bmatrix} u_{fd} \\ u_{kd} \\ u_{kq} \end{bmatrix} \qquad \#2$$

The quantities that are considered are current, voltage and flux linkage. The phase quantities have the classic interpretation. The rotor quantities warrant some explanation. The field winding is assumed to be excited with dc current. The damper windings on the rotor are described by assuming that the actual damper windings can be considered to consist of two parts. The first part is an equivalent winding which is in alignment with the field winding. The second part is an equivalent winding which is in quadrature with the field winding.

Modelling the damper windings as two windings in quadrature is classic. The validity of such a model is not usually discussed quantitatively, references [1] - [6]. Reference [1], page 384, states, "It should be emphasized that representation of induced direct-axis rotor currents by a single circuit is an approximation to the actual situation, in which there are a number of conducting paths. However, this approximation has been found to be quite valid in many cases and is generally used to describe machine behavior." Reference [2], pages 212-214, discusses the use of equivalent damper windings. In fact, reference [2] indicates that a model with two windings in the quadrature-axis comprises a more complete model of synchronous machines than a model with just one. Reference [2] goes on to discuss when simplifications of the two quadrature damper windings are appropriate. It states, "Also it is necessary, in most cases, to include all three damper windings in order to portray adequately the transient characteristics of the stator variables and the electromagnetic torque of solid iron rotor machines [2]." Reference [2]'s reference [2] is a paper by the author on the subject of solid iron rotor machines. Reference [3] acknowledges the role of damper windings but considers their effect negligible on the major dynamics of the synchronous machine. Reference [4] treats damper windings similarly. References [5] and [6] model the rotor as a single field winding and equivalent direct and quadrature-axis damper windings. Hence, based on the discussion within the references, the damper windings are modelled using a rotor d-axis equivalent winding and a rotor q-axis equivalent winding. That some of the transient nuances of solid iron rotor machines will not be preserved must be recognised.

In the way of intuitively (physically) justifying the two windings-in-quadrature approach to the damper windings, the physics of the rotor's conductive paths are likened to leakage flux paths in the stator windings. Describing the myriad leakage inductances by a single leakage inductance has proven to be a relatively accurate means of accounting for leakage flux. Likewise, for the purpose of developing a simple model for use in an object oriented simulation, it is entirely appropriate to describe the myriad of rotor conductor paths as two windings in quadrature. If the application of the simulation requires a more accurate model, then the more accurate, and complex, model can be incorporated. For now, the objective is a simple model that allows the most important dynamics to be captured and used in an object oriented simulation.

This chapter will develop the terms of equation (#1), specifically the rotor angle dependence. Equation (#1) is given in "machine variables" or the variables that represent the physical inputs to and physical extent of the actual machine.

2.2.2.1 Stator Inductances

The six phase windings in the stator are taken to be symmetrical windings. Further, to be wholly general, both round rotor and salient pole rotors are considered. The dependence of inductances on the rotor's position uses the electrical rotor angle, θ_{re}. The electrical rotor angle is simply the mechanical rotor angle multiplied by the number of pole pairs.

$$\underline{\mathbf{L}}_{ph} = \begin{bmatrix} L_{aa}(\theta_{re}) & L_{ab}(\theta_{re}) & L_{ac}(\theta_{re}) & L_{ad}(\theta_{re}) & L_{ae}(\theta_{re}) & L_{af}(\theta_{re}) \\ L_{ba}(\theta_{re}) & L_{bb}(\theta_{re}) & L_{bc}(\theta_{re}) & L_{bd}(\theta_{re}) & L_{be}(\theta_{re}) & L_{bf}(\theta_{re}) \\ L_{ca}(\theta_{re}) & L_{cb}(\theta_{re}) & L_{cc}(\theta_{re}) & L_{cd}(\theta_{re}) & L_{ce}(\theta_{re}) & L_{cf}(\theta_{re}) \\ L_{da}(\theta_{re}) & L_{db}(\theta_{re}) & L_{dc}(\theta_{re}) & L_{dd}(\theta_{re}) & L_{de}(\theta_{re}) & L_{df}(\theta_{re}) \\ L_{ea}(\theta_{re}) & L_{eb}(\theta_{re}) & L_{ec}(\theta_{re}) & L_{ed}(\theta_{re}) & L_{ee}(\theta_{re}) & L_{ef}(\theta_{re}) \\ L_{fa}(\theta_{re}) & L_{fb}(\theta_{re}) & L_{fc}(\theta_{re}) & L_{fd}(\theta_{re}) & L_{fe}(\theta_{re}) & L_{ff}(\theta_{re}) \end{bmatrix} \qquad \#3$$

This representation of the stator winding inductances is general. It should be acknowledged that the matrix will be symmetrical about the diagonal. Hence, there are 21 distinct inductances in the matrix. The form that these inductances take depends upon whether the rotor is round or possesses saliency. Both will now be considered.

2.2.2.1.1 Round Rotor Stator Phase Inductances

The stator inductance matrix for a round rotor (smooth air-gap) machine is shown below. It builds upon the work of references [7] and [2]. It is important to realise that these inductances are based upon consideration of only the fundamental component of the air-gap MMF.

$$\underline{\mathbf{L}}_{ph} = \begin{bmatrix} L_{a0}+L_{al} & L_{a0}\cos(\beta) & -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}+\beta\right) & -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}-\beta\right) \\ L_{a0}\cos(\beta) & L_{a0}+L_{al} & L_{a0}\cos\left(\frac{2\pi}{3}-\beta\right) & -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}+\beta\right) & -\frac{1}{2}L_{a0} \\ -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}-\beta\right) & L_{a0}+L_{al} & L_{a0}\cos(\beta) & -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}+\beta\right) \\ L_{a0}\cos\left(\frac{2\pi}{3}+\beta\right) & -\frac{1}{2}L_{a0} & L_{a0}\cos(\beta) & L_{a0}+L_{al} & L_{a0}\cos\left(\frac{2\pi}{3}-\beta\right) & -\frac{1}{2}L_{a0} \\ -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}+\beta\right) & -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}-\beta\right) & L_{a0}+L_{al} & L_{a0}\cos(\beta) \\ L_{a0}\cos\left(\frac{2\pi}{3}-\beta\right) & -\frac{1}{2}L_{a0} & L_{a0}\cos\left(\frac{2\pi}{3}+\beta\right) & -\frac{1}{2}L_{a0} & L_{a0}\cos(\beta) & L_{a0}+L_{al} \end{bmatrix} \quad \#4$$

2.2.2.1.2 Salient Rotor Stator Phase Inductances

Analysis of a salient rotor machine is easier if the air-gap can be characterised by some functional form. Reference [2] offers a suitable method for characterising a salient rotor air-gap. Equation (#5) shows this relationship for the air-gap distance, g. ϕ_r is the angular displacement about the rotor relative to the magnetic axis of the field winding.

$$g(\phi_r) = \frac{1}{\alpha_1 + \alpha_2 \cos(2\phi_r)} \quad \#5$$

The stator winding inductances for a general, round rotor or salient pole, six phase machine are shown below. The precise value of each of the "L" terms depends upon the structure of the rotor and the windings of the stator. Calculating these values is relatively straightforward. The results follow the overall stator inductance matrix.

$$\underline{\mathbf{L}}_{ph}=\begin{bmatrix}
L_{sl}+L_{so}+L_{s2}\cos 2\theta_{ro} & \cos(\beta)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}\right) & \cos\left(\frac{2\pi}{3}+\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}\right) & \cos\left(\frac{2\pi}{3}-\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}-\frac{\beta}{2}\right) \\
\cos(\beta)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\beta}{2}\right) & L_{sl}+L_{so}+L_{s2}\cos 2(\theta_{ro}-\beta) & \cos\left(\frac{2\pi}{3}-\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}-\beta\right) & \cos\left(\frac{2\pi}{3}+\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}-\beta\right) \\
-\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}\right) & \cos\left(\frac{2\pi}{3}-\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}-\frac{\beta}{2}\right) & L_{sl}+L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{2\pi}{3}\right) & \cos(\beta)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{2\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2(\theta_{ro}+\pi) & \cos\left(\frac{2\pi}{3}+\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\pi-\frac{\beta}{2}\right) \\
\cos\left(\frac{2\pi}{3}+\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{\pi}{3}-\beta\right) & \cos(\beta)L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{2\pi}{3}-\frac{\beta}{2}\right) & L_{sl}+L_{so}+L_{s2}\cos 2\left(\theta_{ro}-\frac{2\pi}{3}-\beta\right) & \cos\left(\frac{2\pi}{3}-\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\pi-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2(\theta_{ro}+\pi-\beta) \\
-\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}\right) & \cos\left(\frac{2\pi}{3}+\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2(\theta_{ro}+\pi) & \cos\left(\frac{2\pi}{3}-\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\pi-\frac{\beta}{2}\right) & L_{sl}+L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{2\pi}{3}\right) & \cos(\beta)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{2\pi}{3}-\frac{\beta}{2}\right) \\
\cos\left(\frac{2\pi}{3}-\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{\pi}{3}-\beta\right) & \cos\left(\frac{2\pi}{3}+\beta\right)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\pi-\frac{\beta}{2}\right) & -\frac{1}{2}L_{so}+L_{s2}\cos 2(\theta_{ro}+\pi-\beta) & \cos(\beta)L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{2\pi}{3}-\frac{\beta}{2}\right) & L_{sl}+L_{so}+L_{s2}\cos 2\left(\theta_{ro}+\frac{2\pi}{3}-\beta\right)
\end{bmatrix}$$

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The three inductances that appear in the inductance matrix are L_{al}, L_{ao}, and L_{a2}. L_{al} represents the leakage inductance of a stator phase winding. Whereas the windings are symmetrical, the leakage inductance of each of the phase windings can be said to be equal to that of a-phase, hence the subscript a. L_{ao} is the component of inductance due the stator winding itself and a constant air-gap. L_{a2} is the magnitude of the component of inductance which varies with the rotor position.

$$L_{ao} = \left(\frac{N_s}{2}\right)^2 \pi\mu_o r l \alpha_1 \qquad \#7$$

$$L_{a2} = \frac{1}{2}\left(\frac{N_s}{2}\right)^2 \pi\mu_o r l \alpha_2 \qquad \#8$$

N_s is the equivalent stator windings per phase. r is the air-gap radius. l is the air-gap axial length. α_1 and α_2 come from equation (#5). Note, if L_{a2} is zero (more specifically α_2 in equation (#5) is identically zero), then the effect of saliency is removed and the resulting inductance matrix matches the inductance matrix for a round rotor machine.

These inductances are used to develop the fluxes for each of the six phases. These fluxes are, in turn, used to find the torque and in the voltage equations. Of note, these fluxes represent only the contributions from the stator windings, equation (#9). The mutual fluxes with the rotor windings will be developed in a later section. Furthermore, these inductances represent only the inductances which contribute to the fundamental component of the Fourier series expansion describing the air-gap MMF.

$$\underline{\lambda}_{ph} = \underline{\mathbf{L}}_{ph} \cdot \underline{i}_{ph} \qquad \#9$$

2.2.2.2 Rotor Inductances

The three circuits which are presumed to represent the rotor are the field winding and the two equivalent circuits-in-quadrature, representing the damper windings. The rotor inductances are shown below. As with the stator windings, the precise values of the inductances depend upon the structure of the rotor and the nature of the field and damper windings themselves.

$$\underline{\mathbf{L}}_R = \begin{bmatrix} L_{fl} + L_f & \left(\frac{N_{kd}}{N_f}\right) L_f & 0 \\ \left(\frac{N_{kd}}{N_f}\right) L_f & L_{kdl} + \left(\frac{N_{kd}}{N_s}\right)^2 (L_{ao} + L_{a2}) & 0 \\ 0 & 0 & L_{kql} + \left(\frac{N_{kq}}{N_s}\right)^2 (L_{ao} - L_{a2}) \end{bmatrix} \qquad \#10$$

N_f, N_{kd}, N_{kq} are the equivalent turns per winding for the field, equivalent direct-axis damper and equivalent quadrature-axis damper windings respectively. Subscripts l denote the leakage inductances. The value of the field winding self inductance is shown in equation (#11).

$$L_f = \left(\frac{N_f}{2}\right)^2 \pi \mu_o r l \left(\alpha_1 + \frac{\alpha_2}{2}\right) \qquad \#11$$

Whereas the field winding and the equivalent direct-axis winding of the damper winding are aligned, their fluxes are coupled. On the other hand, the equivalent quadrature-axis winding of the damper winding is perpendicular to the other windings. Furthermore, the rotor does not experience any time-varying inductances due to saliency in the stator. (It is assumed the stator has no saliency.)

2.2.2.3 Air-Gap Inductances

To determine the stator to rotor mutual inductances, the first requirement is to establish a reference for angular displacements. In keeping with reference [6]'s use of reference directions, the angle θ_{re} will represent the angular displacement of the field winding's magnetic axis from the phase-a magnetic axis. The mutual inductances are shown below. As with the stator and rotor inductances, the precise values of the inductances depend upon the machine geometry and winding configurations.

$$\underline{\mathbf{M}} = \begin{bmatrix} M\cos\theta_{re} & L_{akd}\cos\theta_{re} & -L_{akq}\sin\theta_{re} \\ M\cos(\theta_{re}-\beta) & L_{akd}\cos(\theta_{re}-\beta) & -L_{akq}\sin(\theta_{re}-\beta) \\ M\cos\left(\theta_{re}-\frac{2\pi}{3}\right) & L_{akd}\cos\left(\theta_{re}-\frac{2\pi}{3}\right) & -L_{akq}\sin\left(\theta_{re}-\frac{2\pi}{3}\right) \\ M\cos\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) & L_{akd}\cos\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) & -L_{akq}\sin\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) \\ M\cos\left(\theta_{re}+\frac{2\pi}{3}\right) & L_{akd}\cos\left(\theta_{re}+\frac{2\pi}{3}\right) & -L_{akq}\sin\left(\theta_{re}+\frac{2\pi}{3}\right) \\ M\cos\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) & L_{akd}\cos\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) & -L_{akq}\sin\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) \end{bmatrix} \qquad \#12$$

Under the assumption that the stator windings are symmetrical and the stator has no saliency, the magnitudes of the mutual inductances, M, L_{akd}, and L_{akq}, are constant. Furthermore, the magnitudes are the same for each of the six phases.

$$M = \left(\frac{N_f}{N_s}\right)(L_{ao} + L_{a2}) \qquad \#13$$

$$L_{akd} = \left(\frac{N_{kd}}{N_s}\right)(L_{ao} + L_{a2}) \qquad \#14$$

$$L_{akq} = \left(\frac{N_{kq}}{N_s}\right)(L_{ao} - L_{a2}) \qquad \#15$$

2.2.3 Voltage Equations in Machine Variables

The stator winding voltage equations reflect the resistance of the stator windings and the time-varying flux linkages. Furthermore, wye connected windings are assumed.

$$\underline{v}_{ph} = \underline{\mathbf{r}}_s \cdot \underline{i}_{ph} + \frac{d}{dt}\{\underline{\lambda}_{ph}\}$$

$$= \underline{\mathbf{r}}_s \cdot \underline{i}_{ph} + \frac{d}{dt}\{\underline{\mathbf{L}}_{ph} \cdot \underline{i}_{ph} + \underline{\mathbf{M}} \cdot \underline{i}_R\}$$

$$= \underline{\mathbf{r}}_s \cdot \underline{i}_{ph} + \frac{d}{dt}\{\underline{\mathbf{L}}_{ph}\} \cdot \underline{i}_{ph} + \underline{\mathbf{L}}_{ph} \cdot \frac{d}{dt}\{\underline{i}_{ph}\} + \frac{d}{dt}\{\underline{\mathbf{M}}\} \cdot \underline{i}_R + \underline{\mathbf{M}} \cdot \frac{d}{dt}\{\underline{i}_R\}$$

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Two items of note appear in the preceding equation. The structure of the resistance matrix is shown below. It assumes balanced stator windings. The resistance per phase is r_s. Secondly, the $\underline{\mathbf{L}}_{ph}$ and $\underline{\mathbf{M}}$ matrices depend upon the rotor's angular displacement. The rotor's displacement varies with time.--It is **usually** a periodic function. Hence, the two matrices have time derivatives that must be considered. A constant rotor speed is not assumed here. The phase currents are sinusoidally varying in time; therefore, they too have time derivatives. The rotor currents, although the field current is "dc", can have time varying values during transients, which is where the thrust of this research lies.

$$\underline{\mathbf{r}}_s = r_s \cdot \underline{\mathbf{I}} \qquad \#17$$

The rotor circuits' voltage equations are shown below.

$$\underline{v}_R = \underline{\mathbf{r}}_r \cdot \underline{i}_R + \frac{d}{dt}\{\underline{\lambda}_R\}$$

$$= \underline{\mathbf{r}}_r \cdot \underline{i}_R + \frac{d}{dt}\{\underline{\mathbf{M}}^T \cdot \underline{i}_{ph} + \underline{\mathbf{L}}_R \cdot \underline{i}_R\}$$

$$= \underline{\mathbf{r}}_r \cdot \underline{i}_R + \frac{d}{dt}\{\underline{\mathbf{M}}^T\} \cdot \underline{i}_{ph} + \underline{\mathbf{M}}^T \cdot \frac{d}{dt}\{\underline{i}_{ph}\} + \underline{\mathbf{L}}_R \cdot \frac{d}{dt}\{\underline{i}_R\} \qquad \#18$$

$$\underline{\mathbf{r}}_R = \begin{bmatrix} r_f & 0 & 0 \\ 0 & r_{kd} & 0 \\ 0 & 0 & r_{kq} \end{bmatrix} \qquad \#19$$

Taken together, in machine variables, the voltage equations of the six phase synchronous machine are shown below.

$$\begin{bmatrix} \underline{v}_{ph} \\ \underline{v}_R \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{r}}_s & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{r}}_r \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_R \end{bmatrix} + \frac{d}{dt}\left\{\begin{bmatrix} \underline{\mathbf{L}}_{ph} & \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^T & \underline{\mathbf{L}}_R \end{bmatrix}\right\} \cdot \begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_R \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{L}}_{ph} & \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^T & \underline{\mathbf{L}}_R \end{bmatrix} \cdot \frac{d}{dt}\left\{\begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_R \end{bmatrix}\right\} \qquad \#20$$

2.2.4 Electromagnetic Torque in Machine Variables

The description of electromagnetic torque is developed using the standard coenergy approach. The system is mechanically assembled with electrical inputs equal to zero. Then, the electrical variables are allowed to reach their values one at a time.

$$W_{m}' = \sum_{j=1}^{N}\left[\int_{0}^{i_j} \lambda_j(i_1, \cdots, i_{j-1}, \tilde{i}_j, 0, \cdots, 0; \theta_{rm})\, d\tilde{i}_j\right] \qquad \#21$$

When this sum of integrals is evaluated, the expression shown below results. It describes the electromagnetic torque. Of note, no further simplification is possible unless one assumes balanced currents. Doing so would limit the applicability of this expression in a simulation environment.

$$T^{e} = \underline{i}_{ph}^{T} \cdot \underline{Q} \cdot \underline{i}_{ph} + \underline{i}_{ph}^{T} \cdot \underline{Q}' \cdot \underline{i}_{R} \qquad \#22$$

$$\underline{Q} = \begin{bmatrix} -L_{a2}\sin 2\theta_{re} & -2L_{a2}\sin 2\left(\theta_{re}-\frac{\beta}{2}\right) & -2L_{a2}\sin 2\left(\theta_{re}-\frac{\pi}{3}\right) & -2L_{a2}\sin 2\left(\theta_{re}-\frac{\pi}{3}-\frac{\beta}{2}\right) & -2L_{a2}\sin 2\left(\theta_{re}+\frac{\pi}{3}\right) & -2L_{a2}\sin 2\left(\theta_{re}+\frac{\pi}{3}-\frac{\beta}{2}\right) \\ 0 & -L_{a2}\sin 2(\theta_{re}-\beta) & -2L_{a2}\sin 2\left(\theta_{re}-\frac{\pi}{3}-\frac{\beta}{2}\right) & -2L_{a2}\sin 2\left(\theta_{re}-\frac{\pi}{3}-\beta\right) & -2L_{a2}\sin 2\left(\theta_{re}+\frac{\pi}{3}-\frac{\beta}{2}\right) & -2L_{a2}\sin 2\left(\theta_{re}+\frac{\pi}{3}-\beta\right) \\ 0 & 0 & -L_{a2}\sin 2\left(\theta_{re}-\frac{2\pi}{3}\right) & -2L_{a2}\sin 2\left(\theta_{re}-\frac{2\pi}{3}-\frac{\beta}{2}\right) & -2L_{a2}\sin 2(\theta_{re}+\pi) & -2L_{a2}\sin 2\left(\theta_{re}+\pi-\frac{\beta}{2}\right) \\ 0 & 0 & 0 & -L_{a2}\sin 2\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) & -2L_{a2}\sin 2\left(\theta_{re}+\pi-\frac{\beta}{2}\right) & -2L_{a2}\sin 2(\theta_{re}+\pi-\beta) \\ 0 & 0 & 0 & 0 & -L_{a2}\sin 2\left(\theta_{re}+\frac{2\pi}{3}\right) & -2L_{a2}\sin 2\left(\theta_{re}+\frac{2\pi}{3}-\frac{\beta}{2}\right) \\ 0 & 0 & 0 & 0 & 0 & -L_{a2}\sin 2\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) \end{bmatrix}$$

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$$\underline{Q}' = \begin{bmatrix} -M\sin(\theta_{re}) & -L_{akd}\sin(\theta_{re}) & -L_{akq}\cos(\theta_{re}) \\ -M\sin(\theta_{re}-\beta) & -L_{akd}\sin(\theta_{re}-\beta) & -L_{akq}\cos(\theta_{re}-\beta) \\ -M\sin\left(\theta_{re}-\frac{2\pi}{3}\right) & -L_{akd}\sin\left(\theta_{re}-\frac{2\pi}{3}\right) & -L_{akq}\cos\left(\theta_{re}-\frac{2\pi}{3}\right) \\ -M\sin\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) & -L_{akd}\sin\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) & -L_{akq}\cos\left(\theta_{re}-\frac{2\pi}{3}-\beta\right) \\ -M\sin\left(\theta_{re}+\frac{2\pi}{3}\right) & -L_{akd}\sin\left(\theta_{re}+\frac{2\pi}{3}\right) & -L_{akq}\cos\left(\theta_{re}+\frac{2\pi}{3}\right) \\ -M\sin\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) & -L_{akd}\sin\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) & -L_{akq}\cos\left(\theta_{re}+\frac{2\pi}{3}-\beta\right) \end{bmatrix} \qquad \#24$$

2.2.5 Synchronous Machine Equations in Machine Variables

Now, all of the equations which describe the behaviour, steady state and dynamic, will be given in machine variables. This is merely a collection of the equations given to this point.

$$\begin{bmatrix} \underline{v}_{ph} \\ \underline{v}_{R} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{r}}_{s} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{r}}_{r} \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_{R} \end{bmatrix} + \frac{d}{dt}\left\{\begin{bmatrix} \underline{\mathbf{L}}_{ph} & \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^{T} & \underline{\mathbf{L}}_{R} \end{bmatrix}\right\} \cdot \begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_{R} \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{L}}_{ph} & \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^{T} & \underline{\mathbf{L}}_{R} \end{bmatrix} \cdot \frac{d}{dt}\left\{\begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_{R} \end{bmatrix}\right\} \qquad \#25$$

$$T^{e} = \underline{i}_{ph}^{T} \cdot \underline{\mathbf{Q}} \cdot \underline{i}_{ph} + \underline{i}_{ph}^{T} \cdot \underline{\mathbf{Q}}' \cdot \underline{i}_{R} \qquad \#26$$

$$T^{e} - T_{load} = J\dot{\omega}_{rm} + B\dot{\theta}_{rm} + K(\theta_{rm} - \theta_{equil}) \qquad \#27$$

$$\theta_{rm} = \int_{0}^{t} \omega_{rm}(\tau)d\tau \quad \text{or} \quad \omega_{rm}(t) = \dot{\theta}_{rm}(t) \qquad \#28$$

These represent twelve equations. Assuming that nine variables are known, suppose the voltages at the terminals of the machine can be specified, then equations (#25, #26 and #27) can be used to solve for the remaining twelve variables, the nine phase currents, electromagnetic torque, instantaneous rotor speed and rotor angle. It is vital to realise that no steady state assumptions have been made in equation (#28).

The last step is to normalise the equations shown above. Presumably a power base and voltage base are specified for the system to which the six phase machine will be connected; that is, P_B and V_B are given. For now, consider V_B to be an rms, line-to-neutral voltage. Accordingly, I_B represents an rms line current.

$$I_{B} = \frac{P_{B}}{6V_{B}} \qquad \#29$$

$$Z_{B} = \frac{V_{B}}{I_{B}} \qquad \#30$$

$$T_{B} = \frac{p}{\omega_{B}} P_{B} \qquad \#31$$

p, in equation (#31), is the number of pole-pairs of the machine. ω_B is a base frequency.--It is not necessarily the synchronous frequency. Whereas the field circuit and the damper circuits operate at different voltage and current levels from the stator circuits, the base quantities of the rotor circuit will be different from those of the stator circuit. Several constraints placed upon the structure of the normalised inductance matrix fix the values of the rotor base quantities. Shown below is the constraint on the power level.

$$6V_{B}I_{B} = V_{fB}I_{fB}$$

$$= V_{kB}I_{kB} \qquad \#32$$

$$\underline{\mathbf{Z}}_{\mathrm{fkB}} = \begin{bmatrix} \frac{1}{Z_{\mathrm{fB}}} & 0 & 0 \\ 0 & \frac{1}{Z_{\mathrm{kB}}} & 0 \\ 0 & 0 & \frac{1}{Z_{\mathrm{kB}}} \end{bmatrix} \quad \#33$$

$$Z_{\mathrm{fB}} = \frac{V_{\mathrm{fB}}}{I_{\mathrm{fB}}}$$

$$Z_{\mathrm{kB}} = \frac{V_{\mathrm{kB}}}{I_{\mathrm{kB}}} \quad \#34$$

$$\underline{\tilde{\mathbf{L}}} = \begin{bmatrix} \frac{1}{Z_{\mathrm{B}}} \cdot \underline{\mathbf{L}}_{\mathrm{ph}} & \frac{1}{Z_{\mathrm{B}}} \cdot \underline{\mathbf{M}} \\ \underline{\mathbf{Z}}_{\mathrm{fkB}} \cdot \underline{\mathbf{M}}^{\mathrm{T}} & \underline{\mathbf{Z}}_{\mathrm{fkB}} \cdot \underline{\mathbf{L}}_{\mathrm{R}} \end{bmatrix} \quad \#35$$

$$\underline{\tilde{\mathbf{r}}} = \begin{bmatrix} \frac{1}{Z_{\mathrm{B}}} \cdot \underline{\mathbf{r}}_{s} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{Z}}_{\mathrm{fkB}} \cdot \underline{\mathbf{r}}_{\mathrm{R}} \end{bmatrix} \quad \#36$$

$$\underline{\tilde{\mathbf{Q}}} = \left(\frac{\omega_{\mathrm{B}}}{6pZ_{\mathrm{B}}} \right) \cdot \underline{\mathbf{Q}} \quad \#37$$

$$\underline{\tilde{\mathbf{Q}}}' = \left(\frac{\omega_{\mathrm{B}}}{6pV_{\mathrm{B}}} \right) \cdot \begin{bmatrix} I_{\mathrm{fB}} & 0 & 0 \\ 0 & I_{\mathrm{kB}} & 0 \\ 0 & 0 & I_{\mathrm{kB}} \end{bmatrix} \cdot \underline{\mathbf{Q}}' \quad \#38$$

$$\underline{\tilde{\mathbf{q}}} = [\underline{\tilde{\mathbf{Q}}} \ \ \underline{\tilde{\mathbf{Q}}}'] \quad \#39$$

$$H = \frac{J\omega_{\mathrm{B}}}{2pT_{\mathrm{B}}} \quad \#40$$

Using these normalisations, the voltage and torque equations appear as below. It is interesting to consider several aspects of these equations. First of all, no steady state assumption has been made. Hence, the time derivatives of the inductance matrices in the voltage equation do not resolve into multiplication by a constant rotor speed. The dynamical equation, (#43), is made into a first order equation in $\dot{\omega}_{\mathrm{rm}}$ and $\dot{\theta}_{\mathrm{rm}}$; such a substitution requires the addition of equation (#28).

$$\underline{\tilde{v}} = \underline{\tilde{\mathbf{r}}} \cdot \underline{\tilde{i}} + \frac{d}{dt}\{\underline{\tilde{\mathbf{L}}}\} \cdot \underline{\tilde{i}} + \underline{\tilde{\mathbf{L}}} \cdot \frac{d}{dt}\{\underline{\tilde{i}}\} \quad \#41$$

$$\tau^e = \underline{\tilde{i}}_{ph}^T \cdot \underline{\tilde{q}} \cdot \underline{\tilde{i}} \qquad \#42$$

$$\tau^e - \left(\frac{T_{load}}{T_B}\right) = H\left(\frac{p}{\omega_B}\right)\dot{\omega}_{rm} + \left(\frac{B}{T_B}\right)\dot{\theta}_{rm} + \left(\frac{K}{T_B}\right)(\theta_{rm} - \theta_{equil}) \qquad \#43$$

$$\omega_{rm} = \dot{\theta}_{rm} \qquad \#44$$

3 Six Phase dq Transformation (Rotor Coordinates)

3.1 Park's Transformation for Six Phases

The matrix shown below is the transformation matrix for a six phase machine. It is a derivative of Park's transformation for a three phase machine. It converts the phase quantities of the stator into a component in the direction of the rotor field axis, the "d"-axis, and a component in quadrature with the rotor field axis, the "q"-axis. The four additional quantities are stator quantities that do not contribute to a vector that would have any spatial relationship with the rotor field axis at all. These are the "zero" sequence quantities.

$$\underline{T} = \frac{1}{3}\cdot\begin{bmatrix} \cos\theta & \cos(\theta-\beta) & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}-\beta\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}-\beta\right) \\ -\sin\theta & -\sin(\theta-\beta) & -\sin\left(\theta-\frac{2\pi}{3}\right) & -\sin\left(\theta-\frac{2\pi}{3}-\beta\right) & -\sin\left(\theta+\frac{2\pi}{3}\right) & -\sin\left(\theta+\frac{2\pi}{3}-\beta\right) \\ \cos\theta & -\cos(\theta-\beta) & \cos\left(\theta-\frac{2\pi}{3}\right) & -\cos\left(\theta-\frac{2\pi}{3}-\beta\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & -\cos\left(\theta+\frac{2\pi}{3}-\beta\right) \\ -\sin\theta & \sin(\theta-\beta) & -\sin\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta-\frac{2\pi}{3}-\beta\right) & -\sin\left(\theta+\frac{2\pi}{3}\right) & \sin\left(\theta+\frac{2\pi}{3}-\beta\right) \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

#45

This matrix differs somewhat from the matrix presented in reference [7]. To describe the physical significance of this transformational matrix, think of the six phase machine as consisting of two three phase systems displaced spatially from each other. The notion of the transformation is to resolve the stator phase quantities into components aligned with the rotor's field axis or in quadrature with it. The first row in the matrix determines the portion of the two three phase systems which combine to form a component aligned with the rotor's direct-axis. The second row determines the portion of the two three phase systems which combine to form a component in quadrature with the rotor's direct-axis. The third row determines the portion of the two three phase systems which, independently are parallel or anti-parallel to the rotor's direct-axis, but cancel the component of the other so that no component in the rotor's direct-axis exists. Likewise, the fourth row determines the portion of the two three phase systems which, independently are perpendicular to the rotor's direct-axis, but cancel the component of the other so that no component in the rotor's quadrature-axis exists. The fifth and sixth rows determine the components of each of the two three phase systems which are in temporal phase, and, hence, cancel due to their spatial phase yielding no component in the rotor's direct or quadrature axis.

This interpretation of the transformation matrix agrees with that of reference [7] except for rows number three and four. Reference [7] chooses to neglect this possible phase contribution and instead treats reverse sequence quantities. Both the transformation offered here and in reference [7] are nonsingular, and their inverses are virtually their transposes. Hence, both transformations are valid changes of variables. Reference [2], page 136, states, "Although the transformation to the arbitrary reference frame is a change of variables and needs no physical connotation, it is often convenient to visualize the transformation equations as trigonometric relationships between [phase] variables..." The arbitrary reference frame referred to in [2] is a generalisation of the rotor reference frame adopted by this paper and reference [7]. It appears that the transformations offered here and in reference [7] are valid and will provide correct results. Of note though, the dot product of the different rows of the two transformations are not unity. Hence, the transformations are not equal. Some review of the validity of the transformation matrix proposed here is in order; although, the author will defend the one presented. Perhaps a future research paper could concentrate upon all of the different possible six-phase transformation matrices.

The inverse of the transformation matrix is shown below. The determinant of the transformation matrix is a real, non-zero number. It also is important to note that the inverse transformation matrix is the transpose of the transformation matrix itself but for a normalisable scaling factor. Whereas $\underline{\mathbf{T}}^{-1} \approx \underline{\mathbf{T}}^{\mathrm{T}}$, then it is true that the transformation matrix is orthogonal.--That is, all six rows comprise vectors perpendicular to each of the other five.

$$\underline{\mathbf{T}}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & \cos\theta & -\sin\theta & 1 & 0 \\ \cos(\theta-\beta) & -\sin(\theta-\beta) & -\cos(\theta-\beta) & \sin(\theta-\beta) & 0 & 1 \\ \cos\left(\theta-\frac{2\pi}{3}\right) & -\sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & -\sin\left(\theta-\frac{2\pi}{3}\right) & 1 & 0 \\ \cos\left(\theta-\frac{2\pi}{3}-\beta\right) & -\sin\left(\theta-\frac{2\pi}{3}-\beta\right) & -\cos\left(\theta-\frac{2\pi}{3}-\beta\right) & \sin\left(\theta-\frac{2\pi}{3}-\beta\right) & 0 & 1 \\ \cos\left(\theta+\frac{2\pi}{3}\right) & -\sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & -\sin\left(\theta+\frac{2\pi}{3}\right) & 1 & 0 \\ \cos\left(\theta+\frac{2\pi}{3}-\beta\right) & -\sin\left(\theta+\frac{2\pi}{3}-\beta\right) & -\cos\left(\theta+\frac{2\pi}{3}-\beta\right) & \sin\left(\theta+\frac{2\pi}{3}-\beta\right) & 0 & 1 \end{bmatrix} \qquad \#46$$

The transformation matrix transforms phase variables into d-q variables.

$$\begin{bmatrix} u_d \\ u_q \\ u_{0d} \\ u_{0q} \\ u_{01} \\ u_{02} \end{bmatrix} = \underline{\mathbf{T}} \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \end{bmatrix} \qquad \#47$$

The inverse transformation matrix returns the d-q variables to phase variables. It is most important to note that the transformation matrix and its inverse are functions of the rotor angle, which is a function of time. Hence, the transformation matrix is different at the end of each new time increment. It is important to note, though, that the transformation matrix does not need to be differentiated or integrated.

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \end{bmatrix} = \underline{\mathbf{T}}^{-1} \cdot \begin{bmatrix} u_d \\ u_q \\ u_{0d} \\ u_{0q} \\ u_{01} \\ u_{02} \end{bmatrix} \qquad \#48$$

3.2 Fluxes in Transformed Variables

All of the machine fluxes are described in equation (#1), which is written here. This equation will now be transformed into d-q quantities and rotor variables. All of the elements of the submatrices are defined and described in section 2.2.2. The transformation matrix is defined in the preceding section.

$$\begin{bmatrix} \underline{\lambda}_{ph} \\ \underline{\lambda}_R \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{L}}_{ph} & \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^T & \underline{\mathbf{L}}_R \end{bmatrix} \begin{bmatrix} \underline{i}_{ph} \\ \underline{i}_R \end{bmatrix} \qquad \#49$$

$$\begin{bmatrix} \underline{\lambda}_{dq} \\ \underline{\lambda}_R \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{T}} \cdot \underline{\mathbf{L}}_{ph} \cdot \underline{\mathbf{T}}^{-1} & \underline{\mathbf{T}} \cdot \underline{\mathbf{M}} \\ \underline{\mathbf{M}}^T \cdot \underline{\mathbf{T}}^{-1} & \underline{\mathbf{L}}_R \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{dq} \\ \underline{i}_R \end{bmatrix} \qquad \#50$$

Each of the four submatrices in the preceding equation are considered. The stator inductances, in $\underline{\mathbf{L}}_{ph}$, depend upon the rotor angle. The six phase transformation seeks to remove this time-varying dependence of inductances on the rotor position. Whereas the round rotor machine is a specific case of the more general salient rotor machine, only the salient rotor equations will be transformed. The air-gap inductance matrices, $\underline{\mathbf{M}}$ and $\underline{\mathbf{M}}^T$, also depend upon rotor angle. The transformation matrix should also remove the rotor angle dependence from these two submatrices. Lastly, the rotor inductances, $\underline{\mathbf{L}}_R$, have no dependence on the rotor angle.--As they are not multiplied by a transformational matrix this quality will not change.

Each of the three submatrix multiplications indicated in equation (#50) was carried out using MACSYMA, reference [8]. The results are shown below. As can be seen, none of the inductances are dependent upon rotor angle. Instead, all of the inductances are constants. Hence, the transformation achieves its goal. Additionally, the d-axis and q-axis quantities are decoupled.

$$\underline{\mathbf{L}}_{\mathrm{dq}} = \underline{\mathbf{T}} \cdot \underline{\mathbf{L}}_{\mathrm{ph}} \cdot \underline{\mathbf{T}}^{-1} = \begin{bmatrix} (3L_{\mathrm{ao}} + L_{\mathrm{al}} + 3L_{\mathrm{a2}}) & 0 & 0 & 0 & 0 & 0 \\ 0 & (3L_{\mathrm{ao}} + L_{\mathrm{al}} - 3L_{\mathrm{a2}}) & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{\mathrm{al}} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{\mathrm{al}} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{\mathrm{al}} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{\mathrm{al}} \end{bmatrix} \qquad \#51$$

$$\underline{\mathbf{M}}_{\mathrm{dq}} = \underline{\mathbf{T}} \cdot \underline{\mathbf{M}} = \begin{bmatrix} M & L_{\mathrm{akd}} & 0 \\ 0 & 0 & L_{\mathrm{akq}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \#52$$

$$\underline{\mathbf{M}}_{\mathrm{qd}} = \underline{\mathbf{M}}^{\mathrm{T}} \cdot \underline{\mathbf{T}}^{-1} = \begin{bmatrix} 3M & 0 & 0 & 0 & 0 & 0 \\ 3L_{\mathrm{akd}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 3L_{\mathrm{akq}} & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \#53$$

$$\begin{bmatrix} \underline{\lambda}_{\mathrm{dq}} \\ \underline{\lambda}_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} (3L_{\mathrm{ao}}+L_{\mathrm{al}}+3L_{\mathrm{a2}}) & 0 & 0 & 0 & 0 & 0 & M & L_{\mathrm{akd}} & 0 \\ 0 & (3L_{\mathrm{ao}}+L_{\mathrm{al}}-3L_{\mathrm{a2}}) & 0 & 0 & 0 & 0 & 0 & 0 & L_{\mathrm{akq}} \\ 0 & 0 & L_{\mathrm{al}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{\mathrm{al}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{\mathrm{al}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{\mathrm{al}} & 0 & 0 & 0 \\ 3M & 0 & 0 & 0 & 0 & 0 & L_{\mathrm{fl}}+L_{\mathrm{f}} & \left(\frac{N_{\mathrm{kd}}}{N_{\mathrm{f}}}\right)L_{\mathrm{f}} & 0 \\ 3L_{\mathrm{akd}} & 0 & 0 & 0 & 0 & 0 & \left(\frac{N_{\mathrm{kd}}}{N_{\mathrm{f}}}\right)L_{\mathrm{f}} & L_{\mathrm{kdl}}+\left(\frac{N_{\mathrm{kd}}}{N_{\mathrm{s}}}\right)^2(L_{\mathrm{ao}}+L_{\mathrm{a2}}) & 0 \\ 0 & 3L_{\mathrm{akq}} & 0 & 0 & 0 & 0 & 0 & 0 & L_{\mathrm{kql}}+\left(\frac{N_{\mathrm{kq}}}{N_{\mathrm{s}}}\right)^2(L_{\mathrm{ao}}-L_{\mathrm{a2}}) \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{\mathrm{dq}} \\ \underline{i}_{\mathrm{R}} \end{bmatrix}$$

#54

3.3 Voltage Equations in Transformed Variables

The voltage equations for the six phase machine are cast into d-q form. In the transformation, the diagonal resistance matrix is premultiplied by the transformation matrix and postmultiplied by the inverse of the transformation matrix. Hence, the diagonal resistance matrix results, now in the d-q form.

$$\begin{bmatrix} \underline{v}_{\mathrm{dq}} \\ \underline{v}_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{r}}_{\mathrm{s}} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{r}}_{\mathrm{R}} \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{\mathrm{dq}} \\ \underline{i}_{\mathrm{R}} \end{bmatrix} + \frac{d}{dt}\left\{ \begin{bmatrix} \underline{\lambda}_{\mathrm{dq}} \\ \underline{\lambda}_{\mathrm{R}} \end{bmatrix} \right\} \qquad \#55$$

The time derivative of flux linkage in equation (#55), with the flux linkages described by equation (#54), is a lot less complicated than the equivalent set of equations in machine variables, equation (#16). For one thing, the inductance matrix in equation (#54) is constant; hence, it comes outside of the time derivative operator. The resulting equations are first order, constant coefficient, linear differential equations in the d-q currents. This is significantly easier to work with than the nonlinear equations, which include "speed voltage" terms, in machine variables. The price paid for this simplification is having to perform the transformation to the d-q variables, then the inverse transformation to recover the machine variables.

$$\begin{bmatrix} \underline{v}_{dq} \\ \underline{v}_{R} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{r}}_{s} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{r}}_{R} \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{dq} \\ \underline{i}_{R} \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{L}}_{dq} & \underline{\mathbf{M}}_{dq} \\ \underline{\mathbf{M}}_{qd} & \underline{\mathbf{L}}_{R} \end{bmatrix} \cdot \frac{d}{dt}\left\{ \begin{bmatrix} \underline{i}_{dq} \\ \underline{i}_{R} \end{bmatrix} \right\} \qquad \text{\#56}$$

3.4 Torque in Transformed Variables

The torque equation is now developed in the d-q variables. The transformation is applied to equation (#22).

$$\mathrm{T}^{e} = \underline{i}_{dq}^{T} \cdot \underline{\mathbf{T}}^{-1^{T}} \cdot \underline{\mathbf{Q}} \cdot \underline{\mathbf{T}}^{-1} \cdot \underline{i}_{dq} + \underline{i}_{dq}^{T} \cdot \underline{\mathbf{T}}^{-1^{T}} \cdot \underline{\mathbf{Q}}' \cdot \underline{i}_{R} \qquad \text{\#57}$$

$$\underline{\mathbf{T}}^{-1^{T}} \cdot \underline{\mathbf{Q}} \cdot \underline{\mathbf{T}}^{-1} = \begin{bmatrix} 0 & 9L_{a2} & 0 & 0 & 0 & 0 \\ 9L_{a2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{\#58}$$

$$\underline{\mathbf{T}}^{-1^{T}} \cdot \underline{\mathbf{Q}}' = \begin{bmatrix} 0 & 0 & -3L_{akq} \\ 3M & 3L_{akd} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{\#59}$$

$$\therefore \mathrm{T}^{e} = 18L_{a2} i_{d} i_{q} + 3M i_{q} i_{fd} + 3L_{akd} i_{q} i_{kd} - 3L_{akq} i_{d} i_{kq} \qquad \text{\#60}$$

3.5 Synchronous Machine Equations in Transformed Variables

To the equations describing the voltages and torque must be added the transformation equations, and the inverse transformations as well.

$$\underline{v}_{dq} = \underline{\mathbf{T}} \cdot \underline{v}_{ph}$$

$$\underline{i}_{dq} = \underline{\mathbf{T}} \cdot \underline{i}_{ph} \qquad \text{\#61}$$

$$\begin{bmatrix} \underline{v}_{dq} \\ \underline{v}_R \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{r}}_s & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{r}}_R \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{dq} \\ \underline{i}_R \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{L}}_{dq} & \underline{\mathbf{M}}_{dq} \\ \underline{\mathbf{M}}_{qd} & \underline{\mathbf{L}}_R \end{bmatrix} \cdot \frac{d}{dt}\left\{ \begin{bmatrix} \underline{i}_{dq} \\ \underline{i}_R \end{bmatrix} \right\} \qquad \#62$$

$$\mathrm{T}^e = 18L_{a2}i_d i_q + 3M i_q i_{fd} + 3L_{akd} i_q i_{kd} - 3L_{akq} i_d i_{kq} \qquad \#63$$

$$\mathrm{T}^e - \mathrm{T}_{load} = J\dot{\omega}_{rm} + B\dot{\theta}_{rm} + K(\theta_{rm} - \theta_{equil}) \qquad \#64$$

$$\omega_{rm} = \dot{\theta}_{rm} \qquad \#65$$

Just as with the machine variables, these equations are normalised. Accordingly, base quantities are assigned, namely base power and voltage. These would be the same quantities given in equations (#29), (#30) and (#31). The normalised, time-invariant inductance matrix is defined in equation (#66).

$$\underline{\tilde{\mathbf{L}}}_{dq} = \begin{bmatrix} \frac{1}{Z_B} \cdot \underline{\mathbf{L}}_{dq} & \frac{1}{Z_B} \cdot \underline{\mathbf{M}}_{dq} \\ \underline{\mathbf{Z}}_{fkB} \cdot \underline{\mathbf{M}}_{qd} & \underline{\mathbf{Z}}_{fkB} \cdot \underline{\mathbf{L}}_R \end{bmatrix} \qquad \#66$$

Applying these base quantities, as well as base torque and per-unitised inertia, yields the following equations.

$$\underline{\tilde{v}}_{dq} = \underline{\mathbf{T}} \cdot \underline{\tilde{v}}_{ph}$$

$$\underline{\tilde{i}}_{dq} = \underline{\mathbf{T}} \cdot \underline{\tilde{i}}_{ph} \qquad \#67$$

$$\begin{bmatrix} \underline{\tilde{v}}_{dq} \\ \underline{\tilde{v}}_R \end{bmatrix} = \underline{\tilde{\mathbf{r}}} \cdot \begin{bmatrix} \underline{\tilde{i}}_{dq} \\ \underline{\tilde{i}}_R \end{bmatrix} + \underline{\tilde{\mathbf{L}}}_{dq} \cdot \frac{d}{dt}\left\{ \begin{bmatrix} \underline{\tilde{i}}_{dq} \\ \underline{\tilde{i}}_R \end{bmatrix} \right\} \qquad \#68$$

$$\tau^e = \left(\frac{6}{2}\right)\left(\frac{1}{p}\right)\left[\frac{\omega_B L_{a2}}{Z_B}\tilde{i}_d\tilde{i}_q + \frac{1}{6}\left(\frac{\omega_B I_{fB} M}{V_B}\tilde{i}_q\tilde{i}_{fd} + \frac{\omega_B I_{kB} L_{akd}}{V_B}\tilde{i}_q\tilde{i}_{kd} - \frac{\omega_B I_{kB} L_{akq}}{V_B}\tilde{i}_d\tilde{i}_{dq}\right)\right] \qquad \#69$$

$$\tau^e - \left(\frac{\mathrm{T}_{load}}{\mathrm{T}_B}\right) = H\left(\frac{p}{\omega_B}\right)\dot{\omega}_{rm} + \left(\frac{B}{T_B}\right)\dot{\theta}_{rm} + \left(\frac{K}{T_B}\right)(\theta_{rm} - \theta_{equil}) \qquad \#70$$

$$\omega_{rm} = \dot{\theta}_{rm} \qquad \#71$$

The resistance matrix in equation (#67) is defined by equation (#36), in section 2.2.5.

4 Conclusion/Recommendation

What are the advantages, if any, of using a d-q analysis in the simulation of transient behaviour of a six phase synchronous machine?

The principle advantage of the d-q analysis arises from the elimination of time-varying inductances. The $\underline{\tilde{\mathbf{L}}}$ matrix in equation (#41) must be recomputed each time step. The $\underline{\tilde{\mathbf{L}}}_{dq}$ matrix in equation (#68) is given at the beginning of the simulation. Furthermore, the $\underline{\tilde{\mathbf{L}}}$ matrix in equation (#41) must also be differentiated each time step. This is not the case for the $\underline{\tilde{\mathbf{L}}}_{dq}$ matrix in equation (#68); it is time invariant. The conclusion - the transformation makes the voltage equations much simpler (linear in d-q variables). The price is having to perform the transformation.--The transformation matrix must be recomputed each time step before applying it to the phase quantities. Computation of and differentiation of a (9x9) matrix is substituted with computation of and multiplication by a (6x6) matrix and its inverse. (The functional form of the inverse is known a priori.) The matrices are stated as being (9x9) and (6x6); however, using a Legendre polynomial waveform approach, the dimensions of the matrices will be larger, a function of the number of terms of the Legendre polynomial being retained.

Elimination of the time-varying inductances also simplifies the electromagnetic torque equations. Consider the complexity of equation (#22) given the two time-variant matrices, $\underline{\mathbf{Q}}$ and $\underline{\mathbf{Q}}'$, in equations (#23) and (#24). As with the inductance matrix in the machine variable representation, these two matrices must be recomputed each time step as they are functions of the rotor angle. The torque equation (#60) is significantly simpler, the improvements and penalties due to the d-q transformation being the same as in the voltage equations.

If a waveform approach is being used in the simulation, is the d-q analysis necessary?

Given a waveform approach, the d-q transformation is not necessary. The phase voltages and currents all have assumed waveforms which are solved for iteratively. The rotor angle and rotor speed also have assumed waveforms. The rotor angle waveform must be substituted into the inductance matrix during the iterative solution of the voltage and current waveforms.--This represents a nonlinearity in the solution of the voltage, current and rotor waveforms. Furthermore, the effect of this nonlinearity is pervasive. All of this is to say that using a waveform approach will work in machine variables; however, if saliency of the rotor is an important part of the model, then the mathematics of the solution become nonlinear and very complicated.

Will the d-q analysis simplify or quicken the required computations?

The d-q transformation certainly simplifies the electrodynamical equations; it makes them linear. Hence, the solution for the transfomed variables should converge very quickly. To the solution rate must also be added the time required to perform the transformation and the inverse transformation. Subsequent research of the author may explore this aspect of the d-q transformation and the waveform approach.

Will the d-q analysis improve the accuracy or rate of convergence of the solution?

The answer to this question has not been quantitatively established. For a linear system, the Legendre polynomial waveform approach converges in one step. Hence, the electrodynamical equations converge very quickly. However, the convergence of the input/output or implicit/explicit variables' solution needs to explored quantitatively.--This is the subsequent research area.

In the way of the preceding questions, what are the underlying principles and assumptions of the d-q analysis?

No assumptions are made with regards to rotor speed or rotor angle. Hence, the d-q analysis is general in the sense that it is valid for steady-state and dynamic analyses. The principle is that the rotor does not see time-varying inductances. Hence, the d-q analysis is quite simply a variable transformation. The physical interpretation of the mathematically valid and permissible transformation is discussed in section 3.1. A useful product of the transformation, a decoupled flux model, is dependent upon the assumptions underlying the form of the inductances used and upon the frame of reference of the transformation of variables.

Would these underpinnings limit the utility of the d-q analysis in simulating large (distant from steady state equilibria) transient effects?

No. The preceding paragraph discusses the answer to this question.

What are the peculiarities of six phase machine applications?

Six-phase machines are very similar to classic three-phase machines. In fact, the symmetric six-phase machine is virtually indistinguishable from a three phase machine. In the discussion of the physical interpretation of the d-q transformation, it was helpful to think of the six-phase machine as being two colocated three-phase machines. Hence, the only peculiarity would be to use a symmetric six-phase machine thinking that it would improve on the characteristics of a three-phase machine, aside from increasing current-carrying capability over a three-phase machine with similar phase windings.

How is the six phase machine different from the more familiar three phase machine? How is it similar?

See the preceding discussion.

What do the differences and similarities portend?

The asymmetric six-phase machine offers harmonic reduction. It is also relatively easy to obtain six-phase power from a three-phase system or generate six-phase power in its own right. The technological risks involved in developing six-phase power systems are significantly less than some other number of phases because six-phase power is so similar to traditional three-phase power.

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