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Functional Decomposition of a Medium Voltage DC Integrated Power System

ABSTRACT

The Navy recently produced a Next Generation Integrated Power System Technology Development Roadmap (NAVSEA 2007) that establishes the Navy's goal of incorporating a Medium Voltage DC (MVDC) Integrated Power System (IPS) in future surface combatants and submarines. Some of the technical challenges in implementing a MVDC IPS include the requirement to develop new fault detection and isolation techniques; the establishment of design methods to ensure system stability with constant power loads; standardized methods for controlling prime movers and sharing loads between power generation modules; and a grounding strategy. This paper explores some of these challenges through a functional decomposition and allocation of those functions to different IPS modules. Several different functional allocations are proposed, compared and contrasted. For example, the functions of fault detection, fault location and fault isolation can be accomplished through the use of time-coordinated circuit breakers as is done in traditional a.c. systems (allocated entirely to the power distribution module), or can be accomplished through a combination of controls on the rectifiers of the power generation modules to limit current, fault location algorithms within the controls of the power distribution module, and controlled switches that are part of the distribution module. Each of these two functional allocations has impact on technology development, interface development, and design methodology. Based on the results of this analysis, recommendations for future architectural efforts in maturing MVDC IPS are presented

INTRODUCTION

The Navy has established a goal of incorporating a Medium Voltage DC (MVDC) Integrated Power System (IPS) in future surface combatants and submarine. A MVDC System differs significantly from traditional Medium Voltage AC Systems. In addressing these differences it would be wise to ensure that the design approach for the MVDC system reflect not only the challenges of developing a working system, but also take advantage of the opportunities that extensive use of power electronics and control offer. This paper explores these opportunities and challenges through a functional decomposition and allocation of the functions expected of an IPS system. These functions include: Power Management, System Stability, Fault Response, Power Quality, Maintenance Support, and System Grounding.

MVDC ARCHITECTURE

The MVDC Architecture used in this paper is shown in Figure 1. This architecture is consistent with the NGIPS Technology Development Roadmap. (NAVSEA 2007) The architecture presumes a zonal power system design as described in (Doerry 2005) and (Doerry 2006). The modules included in Figure 1 are:

PGM-M	Power Generation Module type "M"
PGM-A	Power Generation Module type "A"
PDM-A	Medium Voltage DC Power Distribution Module
PMM	Propulsion Motor Module
PCM-B	Zonal Power Conversion Module

In addition to the modules shown in Figure 1, a Power Control (PCON) Module provides control system functionality. While energy storage is an important element of power management, this paper will not directly address the role of a distinct energy storage module.

More detail on the NGIPS architecture can also be found in (Doerry 2007). Of note, this paper does not address the issues with designing the In-Zone Distribution. Some insight in designing In-Zone Distribution can be found in (Doerry and Fireman 2006)

The exact interface voltages, voltage tolerance, current harmonic limits, etc. have not yet been established. Initially, the bus voltages will likely be centered on the ground reference at about ±3000 VDC (6000 VDC line-to-line). Over time, the bus voltage is anticipated to increase as power electronic device technology matures to enable higher voltages.

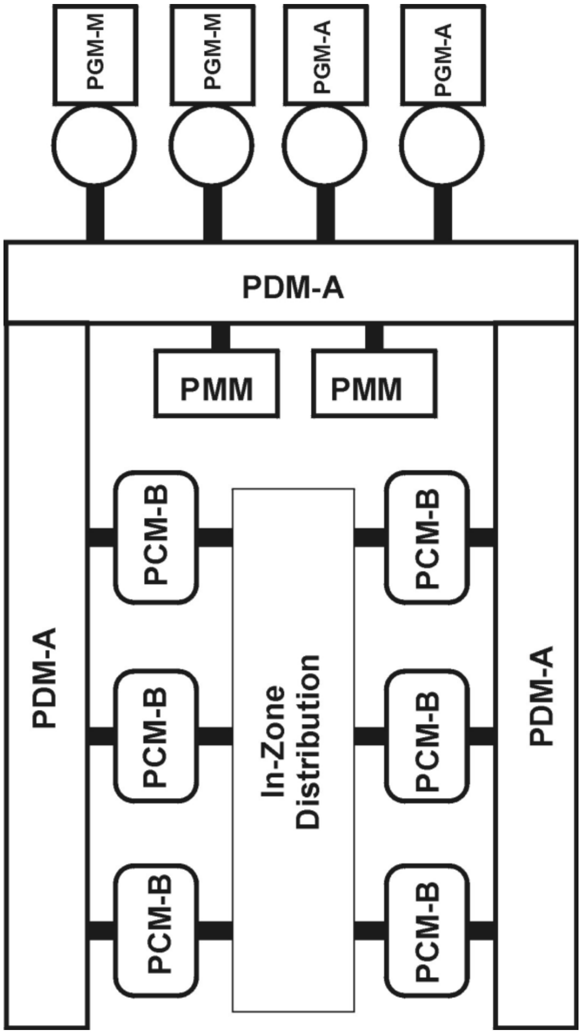


Figure 1: Notional MVDC Architecture

FUNCTIONAL REQUIREMENTS

In designing a power system, a number of functions must be performed to safely generate, transport, and deliver electrical power of the proper quality and continuity needed by the served loads. In dividing a power system into parts, such as that shown in Figure 1, flexibility exists in deciding how to implement and partition these functions to the various parts, or modules.

Because NGIPS is intended to employ an Open Architecture, at some point standards will require development to formalize the responsibility of each module in fulfilling the functions. These common functions include:

- Power Management – Normal Conditions
- Power Management – Quality of Service
- Power Management – Survivability
- System Stability
- Fault Response
- Power Quality
- Maintenance Support
- System Grounding

Power Management – Normal Conditions

Under normal conditions, Power Management ensures that the power system is configured to provide sufficient power to all loads while providing sufficient rolling reserve to address possible step load changes due to pulse loads, large motors starting, and large radars changing modes of operation.

Key to power management is ensuring that the energy used by loads and the energy produced by generation is on average balanced. As shown in Figure 2, for short periods of time, imbalances between generation and loads can be accommodated by Energy Storage. Energy Storage capacity exists within power systems as an inherent feature of the physical hardware. Designers of power systems can also deliberately include Energy Storage capacity to accomplish system functional and performance requirements. Excess generation can be accommodated through Energy Disposal, but with a loss of systems efficiency. Energy storage, whether inherent or by design, has a finite capacity. Overfilling or completely emptying the energy storage will lead to a system failure. Furthermore, a typical power system has many energy storage elements, all with different capacities and rates in which energy can be transferred to and from them. Overfilling or emptying any one of these energy storage elements can lead to a system failure.

Examples of inherent energy storage include the magnetic energy in the windings of a generator, the rotational inertia in the generator, the stored charge in filter capacitors and line capacitance, and magnetic fields of line inductance and filter inductors.

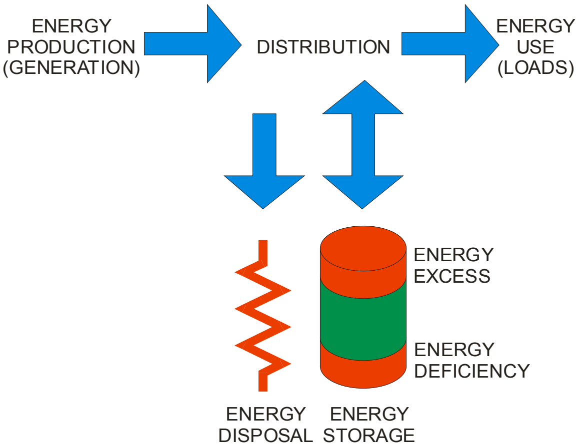


Figure 2: Power Management – Energy Balance

Examples of violating inherent Energy Storage capacities include overspeeding or underspeeding prime movers, voltage collapse on a generator, and insufficient voltage on link capacitors in converters to provide requisite power quality to served loads.

As with Energy Storage, Energy Disposal can be either an inherent aspect of the power system hardware or it can be established by design. Inherent Energy Disposal is usually accomplished through resistive heating of cables or windings. Under fault conditions, Energy Disposal can also manifest itself in arcing. In some motor drives, it is necessary to employ Dynamic Braking Resistors (DBR) as a designed Energy Disposal feature to control the DC link voltage by dissipating regenerative energy from a propulsion motor during crash-backs. For mechanical systems, the inclusion of brakes is also a designed Energy Disposal feature. As with energy storage, each energy disposal method has limits for energy rate (power) and the total energy dissipated, which if exceeded will cause system damage.

Power Management includes dynamics associated with load sharing among the various Power Generation Modules. The power system design must ensure that expected system dynamics do not cause any of the energy storage mechanisms to either overfill or run dry, and that the use of energy disposal is minimized.

In AC systems, real power is shared among paralleled generators through the interactions of the speed governors of the prime movers. Typically, the sharing of real power is implemented through either droop (governors regulate to a lower frequency when providing greater power) or through exchange of power information with dedicated control signals in isosynchronous operation. Reactive power is shared via interaction of voltage regulators. The sharing can once again be implemented by droop (typically based on current) or through dedicated control signals among the voltage regulators paralleled.

In DC systems, particularly those feeding constant power loads, it is not clear how power sharing should be implemented among paralleled generators. The AC solution, based on frequency, will not work in DC systems. Ideally for system reliability, a power sharing method should not require dedicated communication lines.

Under normal operation, one of the critical times for ensuring energy balance is during transients caused by system reconfigurations or by large step changes in load (including pulse power applications). For example, if the power sharing controllers on two paralleled power generation modules are not designed well, during a transient the power will not share properly, which could lead to one of the power generation modules shutting down due to an over-speed or under-speed condition (energy storage via rotational inertia of a generator)

Power Management is very closely tied with system stability. The difference is that Power Management is concerned with establishing the system equilibrium point while System Stability is concerned with ensuring that the system response to a disturbance from that equilibrium point will tend to restore the system to the equilibrium point.

Power Management – Quality of Service

During the initial five minutes following a mismatch in supply and demand of electrical power that can not be accommodated through energy storage or energy disposal, power management sheds sufficient loads that can tolerate five minutes of power interruption to bring supply and demand back into balance while the system reconfigures (including bringing on additional power generation) to enable restoring power to shed loads. Power supply and demand mismatches can occur at the total system level (i.e. total online power generation capacity is less than total ship power load), or can occur on a local basis (i.e. a zone does not have sufficient online power conversion equipment to serve all loads.)

As described in Doerry and Clayton (2005) loads can be placed in one of three proposed Quality of Service (QOS) categories based on how long the load can tolerate an interruption in power service:

- Un-interruptible: Loads that can not tolerate interruptions of 2 seconds or more.

- Short Term Interrupt: Loads that can tolerate interruptions of 2 seconds but can not tolerate interruptions of 5 minutes or more.

- Long Term Interrupt: Loads that can tolerate interruptions of 5 minutes or more.

Un-interruptible loads are typically powered by Un-interruptible power supplies. Short term interrupt loads rely on the Power Management system reconfiguring the electrical distribution system to restore their power. Long term interrupt loads rely on the Power Management system bringing additional power generation capacity online before their power is restored.

Power Management – Survivability

Under the conditions where the power system can not serve all loads, due to either battle damage or equipment failure, power management is required to implement a survivability response. In general, the survivability response is shedding the appropriate loads in the order of their mission priority. Either locally, or across the ship, as necessary, a sufficient number of low priority loads are shed to enable the balance of power availability and power loads.

Survivability also entails restoring power to shed loads if sufficient capacity and connectivity is present and if the load is safe to be re-energized. Because battle damage can result in multiple nearly simultaneous faults (see Figure 3), the algorithms for restoring service must be robust.

Some of the key issues associated with survivability are:

- Determining the health of loads – is it safe to provide power to a given load?

- Determine the health of the different elements of the power system – is it safe to use the given power system element in restoring the power system.

- Isolating unsafe loads and power system elements to prevent further damage or injury. This is particularly important while firefighting and damage control parties are actively combating fires and controlling flooding.

- Determining the optimal configuration of the surviving electrical plant to enable powering the optimal set of surviving ship systems to meet the needs of the crew.

Figure 3: Interior Battle Damage of *Admiral Graf Spee* – scuttled December 1939 following the Battle of the River Plate. – U.S. Navy Photo

System Stability

Within the architecture of Figure 1, all of the loads (PCM and PMM) that directly interact with the sources (PGM) employ power electronic conversion. These power electronic converters regulate their power output and thereby buffer their output from any changes in input bus voltage and, importantly, vice versa. Thus from a power system perspective, all of the loads in the MVDC appear in the short term as constant power loads. Constant power loads have the property of negative incremental resistance. This means that if the voltage to the load increases, its current draw decreases, unlike a conventional load where an increase in system voltage will result in increase current demand. Ensuring system stability with loads having positive incremental resistance is well understood. Unfortunately, assuring stability with loads having negative incremental resistance is much more complex. At high load levels, the problem can be linearized and criteria established. At low load levels however, the highly nonlinear behavior requires special analysis. See Appendix 1 for more details.

Because the MVDC system will behave non-linearly due to the non-linear loads, the classic linear analysis methods for stability are not universally applicable. For power systems, Knazkins (2004) proposes the following pragmatic definition for power system stability:

"Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact."

The challenge for developing a power system technical architecture is being able to specify features and/or behavior of the individual modules such that when integrated into an arbitrary system, the system remains stable. Ideally, the specifications for all modules would be independent; enabling the design and procurement of modules independent of specific ship configuration details. However, the state of understanding of non-linear stability requires specifying unique power system impedances for each module for a given ship application.

Within a MVDC power system, the two principal variables of interest for ensuring stability are:

- Generator Speed: Stability requires that the kinetic energy of the prime mover is neither too low to cause the prime mover to stall/shutdown or the voltage to collapse, nor too high to cause an overspeed induced shutdown or failure. Speed governor dynamics play a critical role in generator speed stability. Of particular concern is that generator speed is not directly observable on the DC bus; only indirectly through the current supplied to the bus. Note that the additional AC power system requirement for the rotor angles of paralleled generators to remain in synchronism is not present in MVDC systems. For fuel cells, the analogous properties are the fuel and oxygen concentrations in the fuel cell stacks.

- Bus Voltage: Stability requires that the stored electro-magnetic energy in the power sources (PGMs and PCMs) are sufficient to keep the bus voltage within power quality range, but not so high as to overload the energy storage capacity. Stability requires the avoidance of resonances with line inductances and bus capacitances and their interaction with non-linear and negative incremental impedance loads. Sudhoff et. al. (2003) describe methods for analyzing voltage stability of DC systems through time-domain simulations, generalized immittance analysis, and polytropic analysis.

Fault Response

Component failures, due either to normal aging of equipment or to external induced damage, may result in the power system being unable to serve its loads, or even to maintain system stability. The fault response function addresses how the power system identifies that a fault has occurred, and then reconfigures the power system to enable continued stable operation while serving the highest number of undamaged loads.

Traditionally, circuit breakers have been used to detect faults based on the magnitude of fault current passing through them. Power sources, such as PGMs and PCMs, are required to provide enough fault current to enable the circuit breakers to detect the fault, but not so much as to exceed the fault current interruption rating of the circuit breakers. Reconfiguration is implemented by time-synchronizing the circuit breakers using time-current coordination curves (see Figure 4). Breakers closer to a source are designed to wait longer to trip for a given level of fault current than breakers further out in the system that serve fewer loads. In this manner, breakers are time-coordinated to ensure the fewest number of loads are isolated. Details on coordinating circuit breakers can be found in the Army’s technical manual TM 5-811-14 (1991). For ring-buses, additional equipment in the form of Multi-Function Monitors (MFMs) are used to aid the coordination of the main bus breakers. Greene (2005) provides a good description of how MFMs are employed in naval ship design.

One of the weaknesses of traditional circuit breakers is their difficulty in identifying and coordinating in response to high impedance arc-faults. These faults do not draw sufficient current to activate the circuit breaker circuitry, but can cause significant damage to equipment as shown in Figure 5. To protect specific equipment from arc faults, the Navy has developed several light sensor, pressure sensor, and thermal ionization sensor based systems to rapidly isolate power in the event of an arc fault within a protected cabinet. For further information on arc faults and arc fault detection, Land (2005) provides an extensive listing of arc fault references.

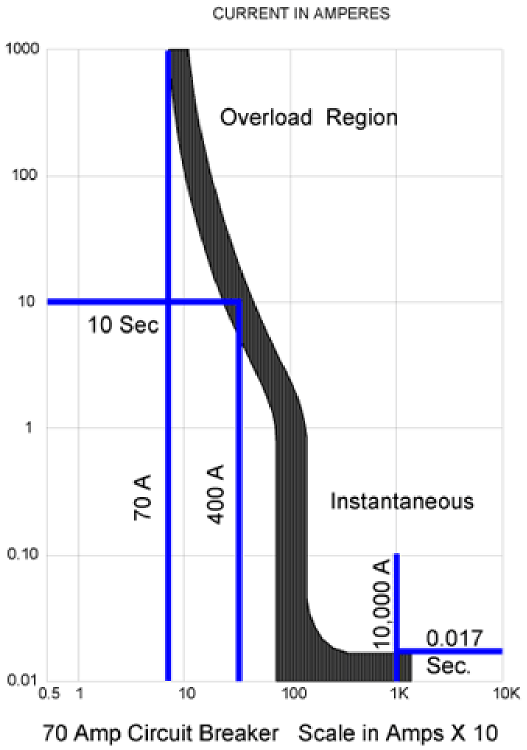


Figure 4: Circuit Breaker Time-Current Coordination Curve (Phillips 2006)

Figure 5: Damage from Arc Fault induced electrical fire (www.arcfault.org)

In MVDC systems, the use of traditional electromechanical circuit breakers is complicated by the need to extinguish the arc once the circuit breaker contactors open. In an AC circuit breaker, the natural zero crossing of the current waveform provides a mechanism for extinguishing the arc and establishing a voltage barrier to prevent the arc from re-striking. DC circuit breakers can not take advantage of the current zero crossing. Hence electromechanical circuit breakers are limited in the amount of DC current they can interrupt. Several manufacturers are developing hybrid DC circuit breakers which use semiconductors to shunt the current when the electro-mechanical breaker opens, thereby eliminating the arc.

Because the MVDC is created by power electronic rectifiers, fault currents can also be limited at the PGM and PCM outputs by controlling the power electronic rectifiers. The challenge confronting system designers of a MVDC system is to understand the behavior of the MVDC system when upstream rectifiers limit current and interrupt current and the rectifiers' criteria for doing so. For more information on the different ways to use power electronics to limit fault current, see Mahajan (2004).

Power Quality

Power Quality refers to the ability of a power system to maintain the voltage waveform within specification values. For DC systems, power quality refers to the voltage tolerance, frequency content of the voltage ripple with respect to the line to line voltages as well as line to ground voltages.

Within power systems, power quality is enabled by placing restrictions on the current waveforms that loads draw. These restrictions include frequency content of the current ripple, maximum step current changes, and maximum power ramp rates for large loads.

In a MVDC power system, the only loads connecting to the MVDC bus are Propulsion Motor Modules, Power Conversion Modules, and potentially a few additional high power loads. Considerable flexibility currently exists to provide standards for Power Quality that can be affordably met by both sources and loads and still provide the requisite power quality and quality of service to end users.

Maintenance Support

Maintenance support addresses system features that enable performing work safely on power system equipment and loads without securing power to a large number of loads. Maintenance support includes methods for isolating equipment, rerouting power around isolated equipment, and ensuring safety through tag-out procedures (see Figure 6). The system design must ensure that control system activity from PCON or internal to modules are in conformance with the tag-out procedures. Maintenance support can also be enabled by incorporating hot-swappable modules where necessary to achieve Quality of Service requirements.

Figure 6: NAVSEA 9890/8 Danger, Do Not Operate Tag

Details on the Navy's tag-out procedures can be found in the NAVSEA Tag-Out Users Manual, S0400-AD-URM-010/TUM (2007).

System Grounding

In general, all power systems are grounded either intentionally through a grounding system, or unintentionally through parasitic capacitances and EMI filters. As detailed by Jacobson and Walker (2007), the benefits of a grounding system include:

- In a fault-free condition, a grounding system establishes a predictable system grounding point and minimizes voltage stress seen by the system elements

- Under transient fault conditions, a grounding system may limit transient voltages applied to insulation systems and equipment.

- Under steady-state fault conditions, a grounding system can facilitate the localization of fault, thereby enabling system reconfiguration to isolate the fault.

If a grounding system is used, accepted practice is grounding each galvanically isolated power system at exactly one point to minimize ground circulating currents. From a system grounding perspective, the functions that must be allocated to modules include:

- Which modules should incorporate galvanic isolation?

- Which modules should incorporate a method for grounding the power system (along with the type of grounding)?

FUNCTIONAL ALLOCATIONS

The previous section described the different functions a power system must perform. In this section, alternative methods are presented highlighting some of the different ways that these functions can be implemented.

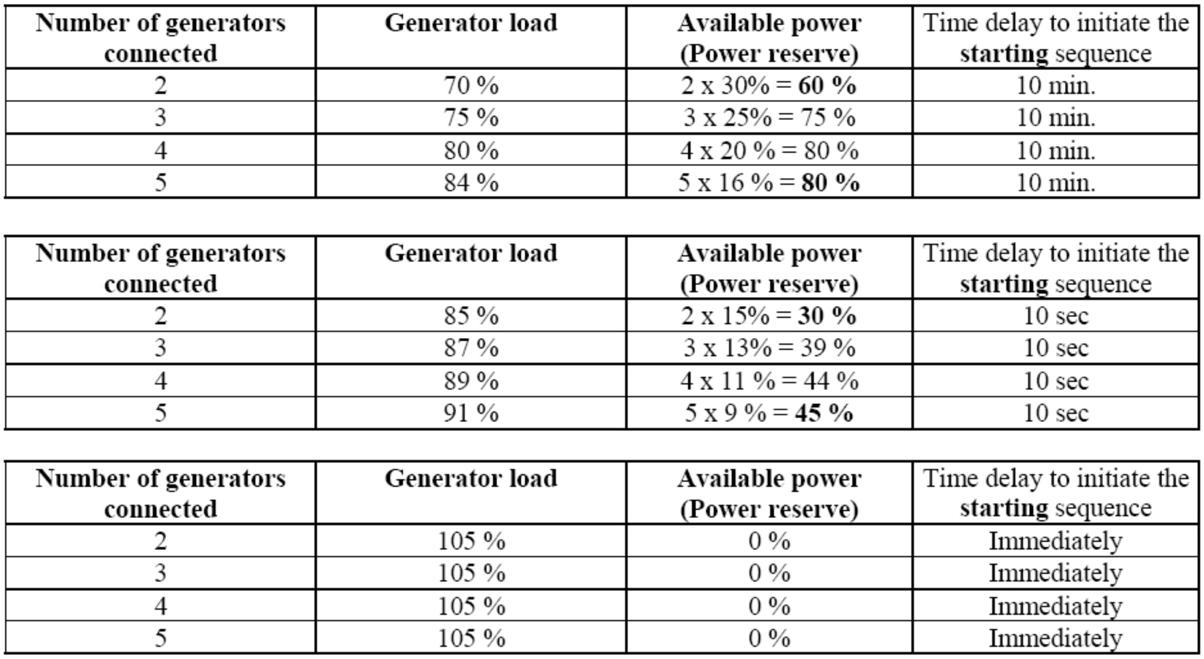
Power Management – Normal Conditions

As stated above, Power Management under Normal Conditions is concerned primarily with determining the optimal amount of power generation that should be online at one time, and the process for ensuring power sharing among the generators. How to implement power sharing without requiring a high bandwidth control signal is still an open question. Two methods for determining the amount of online power generation capacity is presented here.

LOAD DEPENDENT POWER MANAGEMENT MODEL

In the Load Dependent Power Management Model, power control determines how much power generation capacity should be online by monitoring the electrical load over time. If the load approaches some set-point of capacity for a period of time, then additional power generation capacity is brought online. When power demand drops below a different set point for a period of time and more than the minimum number of power generation modules are online (usually 2), power control will take a generator off-line. In a number of implementations, power control will only advise an operator to take a generator off-line – power control will not do so automatically. Also, in many implementations, the time delay before starting or securing a generator will be a function of the load over the time period. For example Table 1 shows an illustrative example for scheduling when to start an additional generator of the same rating based on the amount of load. The minimum amount of Power Reserve (measured in % of a generator rating) depends on the expected variation of power load, on Quality of Service requirements, and on the ability to rapidly shed loads (such as propulsion) should demand increase or an online PGM expectedly shut down. Table 2 shows a corresponding table for determining when to shut down a PGM. For implementation details see Radan (2004).

Table 1: Load Dependent Power Generation Module Starting Example (Radan 2004)



Number of generators connected	**Generator load**	**Available power (Power reserve)**	Time delay to initiate the **starting** sequence
2	70 %	2 x 30% = **60 %**	10 min.
3	75 %	3 x 25% = 75 %	10 min.
4	80 %	4 x 20 % = 80 %	10 min.
5	84 %	5 x 16 % = **80 %**	10 min.

Number of generators connected	**Generator load**	**Available power (Power reserve)**	Time delay to initiate the **starting** sequence
2	85 %	2 x 15% = **30 %**	10 sec
3	87 %	3 x 13% = 39 %	10 sec
4	89 %	4 x 11 % = 44 %	10 sec
5	91 %	5 x 9 % = **45 %**	10 sec

Number of generators connected	**Generator load**	**Available power (Power reserve)**	Time delay to initiate the **starting** sequence
2	105 %	0 %	Immediately
3	105 %	0 %	Immediately
4	105 %	0 %	Immediately
5	105 %	0 %	Immediately

Table 2: Load Dependent Power Generation Module Stopping Example

Number of generators Initially Connected	Generator load (before / after stopping)	Available power (Power reserve) (before / after stopping)	Time delay to initiate **stopping** sequence
3	40% / 60%	180% / 80%	30 min
4	49% / 65%	205%/ 105%	30 min
5	56% / 70%	220% / 120%	30 min
6	63% / 75%	225% / 125%	30 min

RESOURCE MANAGEMENT MODEL

The Resource Management Model extends the concepts of the Load Dependent Power Management Model by dynamically determining the amount of power generation required and the allocation of power to zones. As described by Amy et. al. (1997), each mission system and distributed system has a resource manager. The resource manager, in conjunction with the operator, determine dynamically the priority of each mission system. The mission system resource managers translate these priorities to prioritized resource needs from each distributed system resource manager. In this way, the distributed system resource managers, including PCON, have a dynamic means of establishing power generation needs and priorities.

One advantage of the Resource Management Model is that if implemented well, much of the ship’s control software can be common across all classes of ships. Ship specific data can be centralized in a Ship Properties Resource Manager and a Ship Compartment Information Manager. Furthermore, the control system works identically during normal and damaged conditions – only the resources available to the resource manager changes.

One drawback to the Resource Management Model, unlike the Load Dependent Power Management Model, is that it has never been demonstrated. It’s complexity would require a significant amount of development and testing.

Power Management – Quality of Service

There are at least two methods for ensuring Quality of Service in a shipboard power system: Rolling Reserve and Energy Storage. Both of these methods assume that power system, without the largest capacity power generation module, has sufficient power generation capacity to serve all non-propulsion loads. These methods also assume that upon loss of power generation, long-term interrupt loads can be shed until another power generation module is brought online.

ROLLING RESERVE MODEL

In the rolling reserve model, the power system responds to a shortage in power generation capacity by shedding long-term interrupt loads. Sufficient power generation capacity is maintained online at all times such that uninterruptible and short-term interruptible loads remain powered upon loss of the largest online generator. Long term interrupt loads are restored once additional power generation capacity is brought online.

The rolling reserve model requires at least two power generation modules of sufficient capacity to be online at all times. This can have a significant impact on the minimum rating of a power generation module as well as the efficiency and fuel consumption of the online power generation modules.

ENERGY STORAGE MODEL

In the energy storage model, rolling reserve is not required if the energy storage module has sufficient power and energy capacity to provides sufficient power for the un-interruptible and short-term interruptible loads until additional power generation is brought online.

The Energy Storage Model enables single engine cruise operation, which in many cases can significantly reduce fuel consumption. Reduced operating time of engines can also reduce maintenance requirements. These advantages are offset by the cost and complexity of integrating an Energy Storage Module into the power system.

If energy storage is needed to support specific loads, such as pulse power weapons, then it may prove more economical to use this capacity to implement the Energy Storage Model.

Power Management – Survivability

As stated above, the primary issues with survivability following battle damage are determining what power system components are safe to energize, which loads are safe to energize, and of the safe loads, what is the priority for restoring power. In the Operator-Based Response model, most of the decision making is done by an operator, while in the Agent-Base Response, as much of the decision making as possible is left to PCON.

In both cases, Zonal Design methods are assumed to have been used. This enables either the operator or PCON to focus on the directly damaged zones, since by design, the other zones will survive. Within the damaged zone, much of the survivability effort is directed to emergency loads, and non-redundant mission system loads. Other loads are either non-critical, or are redundant to equipment in undamaged zones.

Additionally, mission priority load shedding would be implemented for either case to ensure demand does not exceed generation capacity. Loads would not be re-energized however, unless doing so is known to be safe.

OPERATOR-BASED RESPONSE MODEL

In the Operator-Based Response Model, PCON is largely used to report the condition of power system equipment and loads. For a number of components, PCON will likely not be able to definitely determine whether a load is safe to energize or not. The operator may employ video cameras, visual checks, or even pre-light-off checks before deciding to energize power equipment or mission loads.

AGENT BASED RESPONSE MODEL

In an Agent Based response model, computer agents for each resource system and mission system are responsible for determining with very high confidence whether a surviving load is safe to energize or not. Once the state of all the equipment under its cognizance is known to a mission system agent, it prioritizes the required resources needed from the resource systems and communicates this information to the resource system agents. The resource system agents, in negotiating with the resource system agents and the mission system agents, determine the optimal set of loads to provide power to.

Although the Agent Based-Response Model adds complexity, and requires much more health monitoring of equipment, this model offers the opportunity to reduce workload demand of the human operators and potentially could restore combat capability much faster – which could be extremely important during combat.

System Stability

Designing a specific power system that is stable over its operating range, while not trivial, is well within the capability of industry today. Less clear, is the capability to define interface and performance requirements for Power Generation Modules, Propulsion Motor Modules, Power Conversion Modules, and Energy Storage Modules such that the module development and procurement can proceed independently of each other, before the final shipboard configuration is known.

Following is a description of two approaches to stability – Linear theory based on small-signal response, and non-linear theory. Each has its advantages and disadvantages; it is not clear which is superior.

LINEAR STABILITY MODEL

The use of Bode stability techniques in power system design is well established (Flower and Hodge (2005), Williams (2004), Gholdston et. al. (2005) and Sudhoff et. al. (2003)). For a simple source and load as shown in Figure 7, the small signal Impedance of the Source (S) and the small signal Admittance[1] (L) of the Load are designed to ensure stability. Flower and Hodge (2005) is representative of the many references that demonstrate that this system is stable if the roots of 1 + SL (where S and L are expressed in terms of their Laplace transform in the form of a ratio of polynomials of the Laplace operator s) all have negative real components.

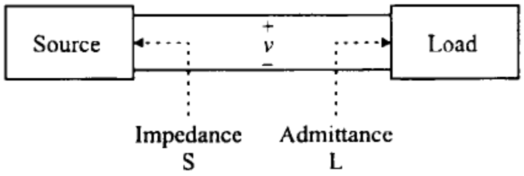


Figure 7: Simple source – load system (Sudhoff et. al. 2003)

If we set $G(s)$ = SL, then the problem is determining if the roots of 1 + $G(s)$ are in the complex left hand plane. This is precisely the problem addressed by Bode diagrams. In Bode diagrams, s is set equal to $j\omega$ where ω is the frequency (measured in radians/sec) and j is the square root of -1. $G(j\omega)$ is now expressed as:

$$G(j\omega) = H(\omega)e^{j\theta(\omega)} \qquad [1]$$

Where $H(\omega)$ is the gain and $\theta(\omega)$ is the phase. Taking the logarithm of [1] results in

$$\ln(G(j\omega)) = \ln(H(\omega)) + j\theta(\omega) \qquad [2]$$

A Bode diagram is a representation of equation [2] and consists of a plot of $H(\omega)$ (in dBs) vs. ω (on a logarithmic scale) and $\theta(\omega)$ (in degrees) vs. ω (also on a logarithmic scale).

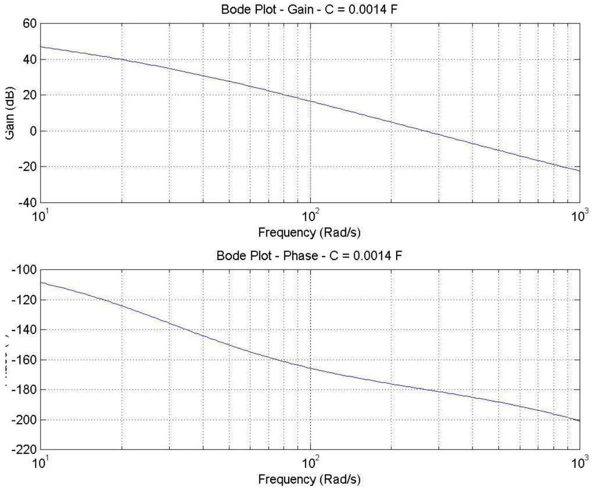


Figure 8: Bode Plot Example (Flower and Hodge 2005)

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[1] Admittance is the reciprocal of Impedance.

For stability, two conditions must exist. First, the gain (in dB) at the frequency where the phase is 180° must be less than 0. The difference between 0 and the gain is called the gain margin. Second, the phase at the frequency where the gain is 0 dB must not be 180°. The difference between this phase and 180° is the phase margin.

In designing a power system, practice is to specify minimum gain and phase margins. Since G(s) = SL is a function of the sources, loads, and the operating point, its not clear how one can create a generalized requirement for sources and loads such that an arbitrary ship configuration at an arbitrary operating point is guaranteed to be stable. However, from Appendix A, bus capacitance is stabilizing. By including the capacitance value as value specified by the customer at time of purchase in a specification for loads, sources and/or a separate module, a design methodology should be feasible for ensuring a particular ship configuration using pre-designed sources and loads is small signal stable. Adding bus capacitance does however, increase the initial fault current on a faulted bus, and the inrush current can cause start up transient problems.

Another issue with this approach to stability is that it is based on linear small-signal stability. System response to large signal disturbances, such as due to a loss of an online power generation module, is not addressed. Furthermore, the impact of non-linearity limits the applicability of linear "small-signal" theory. The magnitude of the non-linear impact should be characterized to ensure that the use of linear theory is warranted and that any non-linear behavior can be accounted for by design robustness2.

NON-LINEAR STABILITY METRICS

Naval electric power systems are, in fact, non-linear, time-varying complex systems which are subject to large signal perturbations. Appropriate stability analysis and design tools must account for this type of system. Nonlinear, or general, stability is a quality of a power system's behavior, its dynamic response, associated with an equilibrium and perturbations relative to that equilibrium. Stability itself is not a monolithic quality. Different forms of stability are described, simple stability, uniform stability, local stability, global stability, asymptotic stability, exponential stability, and combinations of these. While it is likely to be unrealistic to design a naval electric power system which is globally asymptotically stable, it may be realistic to design one which is locally asymptotically stable about equilibria which are important, useful Steady State operating points and that will transition between such in a controlled manner. No single tool exists which can definitively characterize the stability of a naval electric power system. Hence, different tools must be used in a process for designing a locally asymptotically stable time-varying, non-linear complex naval electric power system. The stability metrics for such a system would be the size of the locally asymptotically stable region associated with relevant equilibria.

The following (ideal) process describes the steps required to completely assess the stability of a time-varying, non-linear complex naval electric power system.

1) Accurately model the time-varying, non-linear power system including initial conditions (ICs), system parameters and inputs (U).

2) Determine equilibria, which may be a manifold (infinite equilibria).

3) Determine perturbations about each equilibrium.

4) For each perturbation about each equilibrium, determine the dynamic response of the system and whether it fulfills the required form of stability.

Before stability itself can be assessed in the process above, a significant effort in modeling and solving for the equilibria is necessary. It is important to understand that stability is assessed relative to the system's dynamic behavior associated with the POST-perturbation equilibrium, which may very well be different from the pre-perturbation equilibrium. This may help distinguish between what constitutes a

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[2] Design Robustness is the ability of a system to operate well in off-design conditions.

small-signal perturbation and a large-signal perturbation.

Linear stability tools, linearization techniques, are useful in the process of evaluating identified equilibria. Because the linearization of the power system is accurate only in a neighborhood of an equilibrium, a finding that the linearized system is stable is referred to as "small signal" stability. The following is an important point. A system cannot be stable in a general (nonlinear) sense if it is not stable in a linearized sense about an equilibrium. The converse is NOT true. A system which is stable in a linearized sense about an equilibrium is not necessarily stable in a general (nonlinear) sense. The foregoing suggests that a useful application of linearized model stability analysis would be to use it to relatively quickly "screen" selected equilibria of the time-varying, non-linear power system for unstable equilibria. The significance of these unstable equilibria would then be evaluated as to whether they require power system design modification. The "screening" of the equilibria would also, hopefully, identify equilibria which warrant further, more detailed investigation.

Frequency domain tools require that a 'transfer function' be developed which hopefully would include the effects of ICs, variations in system parameters and inputs (U). Usually, though, 'transfer functions' ignore the effect of initial conditions and variations in system parameters.--Both of which are essential to assessing power system stability. Frequency domain analyses are very useful in assessing "driven" or "forced responses". The forcing functions are usually, implicitly a sinusoidal or exponentially varying amplitude sinusoidal function. The analyses, though, can consider a spectrum of such forced responses at once, not one at a time. This is particularly useful in avoiding "ringing" circuits, a concern in DC portions of a power system.

Time domain tools are necessary to conduct general stability assessments. These are simulation models. They are needed to solve for equilibria. They are linearized for use in linear stability techniques. They are complementary to frequency domain models. They can include the effects of varying initial conditions (ICs), system parameters and inputs (U); they can include both small and large perturbations. Typically, though, a very finite number of such variations can be calculated. Hence, time domain tools used in concert with other tools offer the most reasonable approach to developing locally asymptotically stable time-varying, non-linear complex naval electric power system.

Lyapunov techniques address general stability directly. However, issues associated with necessary and sufficient conditions for stability make the conservativeness of Lyapunov techniques an open question. Perhaps Lyapunov techiques would be best applied to designing real-time stabilizing controllers for distinctly non-linear power systems. Other concepts being developed presently offer potential improvements in time-varying, non-linear stability assessments; these developments, such as polynomial chaos analyses, are largely still at the basic research level.

The foregoing indicates what is necessary with respect to assessing stability. That constitutes the first part of the challenge. The second part is how to stabilize a time-varying, non-linear power system which is not initially stable. One approach has been to use linearizing controllers. These controllers strive to ensure that the time-varying, non-linear portions of the system behave as if they were a linear system. Work has also been published on the use of nonlinear system stabilizing controllers in loads (Glover and Sudhoff 1998). Significant, defining work on describing what form of stability is required needs to be accomplished before a specific approach to system stabilization can be prescribed.

Fault Response

As stated earlier, the fault response function addresses how the power system identifies that a fault has occurred, and then reconfigures the power system to enable continued stable operation while serving the highest number of undamaged loads. A second fault response function is to protect equipment or the cable infrastructure from the effects of the fault

Historically, circuit breakers have been used to detect and isolate electrical faults. The introduction of power electronics offers another opportunity to implement fault response more affordably.

CIRCUIT BREAKER MODEL

As discussed earlier, circuit breakers rely upon large fault currents to properly coordinate the tripping of breakers to localize the isolated section of the power distribution system to the smallest section impacted by the fault. This need for large fault current drives up the cost of power electronics providing power to the circuit breakers. Furthermore, militarized shock hardened circuit breakers can be expensive.

POWER ELECTRONICS MODEL

In the power electronics model, sensors and controls are used to detect and localize faults. Faults are isolated by temporarily de-energizing the distribution system, reconfiguring no-load switches (not capable of interrupting current) to isolate the faulted portion of the distribution system, then re-energizing the system in less than 2 seconds. During this time period, uninterruptible loads must be provided power from an alternate source.

Implementing the Power Electronics Model requires the development of an architecture and design methodology to implement it. Developing open-architecture hardware and software to implement this type of fault detection will likely prove challenging.

Power Quality

The following options deal with how the bus voltage is specified – whether it is set by design to a nominal value (Tight-Tolerance Model) or allowed to vary over a wide range at the command of PCON (Loose Tolerance Model). What is not addressed is the level of voltage ripple at different frequencies that are allowed on voltage and current waveforms. How to determine these allowable limits is still an open question.

TIGHT TOLERANCE MODEL

In a tight tolerance Model, the DC Bus Voltage is maintained within a narrow range around a nominal value, typically ±10%. The advantage of a tight tolerance model is that the loads can be optimized for the specified input voltage. Also, the voltage regulators on paralleled sources may more easily operate independently of each other. The system controls are likely to be simpler than for the loose tolerance model.

LOOSE TOLERANCE MODEL

In a loose tolerance model, the DC Bus Voltage may be varied over a large range, typically 50% to 100% of the maximum voltage. The system will still have a "nominal" value that individual sources will regulate to, but that "nominal" value can be adjusted by PCON to provide the best economy and Quality of Service for the current operating condition. Operating at a lower bus voltage when demand is low can increase the efficiency of propulsion motor modules (particularly the motor drives) and can increase the reliability of power semiconductors and insulation systems. The down side of a loose tolerance model is the complexity of the controls, and potentially increased weight in cable due to the increased current drawn by constant power loads at lower voltages.

Maintenance Support

Historically, maintenance support has been provided via the physical disconnect model. Maintenance procedures have evolved over the years to ensure safety by physically isolating a component from the power system before working on it. Power electronics, if implemented properly, provides an opportunity for electrically isolating equipment in a safe manner at reduced cost and weight.

In both the Physical Disconnect Model and the Control System, Power Electronics Disconnect Model, designing modules to incorporate hot-swappable components to the maximum extent possible can eliminate the need to electrically isolate a module for many maintenance and repair activities.

PHYSICAL DISCONNECT MODEL

In the physical disconnect model, the system designer provides electro-mechanical disconnects in the system to enable electrical isolation of a module or submodule. These disconnects can be in the form of circuit breakers, switches, removable fuses, removable links, or removable sections of cable.

In general, physical disconnects are effective, but add cost and weight to the system. Because they are in series with a component, their reliability can also negatively impact the overall reliability of the system. More electrical connections offer the opportunity for more arcing faults.

CONTROL SYSTEM, POWER ELECTRONICS DISCONNECT MODEL

As power electronics become ubiquitous onboard ship, opportunity exists to use the power electronic semiconductor switches to disconnect power to a module or submodule. The challenge is providing a means for the maintenance personnel to ensure that the power electronics can not inadvertently energize the equipment under maintenance. Consideration for implementing this include physical switches or disconnects on the gate driver boards, and machinery control system support for tagging out components undergoing maintenance or repair.

System Grounding

While there are a number of system grounding issues that must be resolved to ensure a successful MVDC power system, the issue of greatest architectural concern is whether the PCM-B of Figure 1 incorporates galvanic isolation or not.

PCM-B WITH GALVANIC ISOLATION

An advantage of incorporating galvanic isolation in PCM-B is that ensuring that the ground reference of an AC output is easier to establish. Galvanic isolation however, can add size, weight, and cost to the PCM-B. Size, weight and cost effects of adding galvanic isolation can be mitigated through the use of high frequency magnetic circuits."

PCM-B WITHOUT GALVANIC ISOLATION

A PCM-B without galvanic isolation can have a reduced size, weight and potentially cost. The drawback is that AC outputs may have a DC offset with respect to the ground reference that my stress the insulation systems of loads, leading to premature failures.

CONCLUSIONS

This paper explored some of the challenges of implementing a MVDC integrated power system through a functional decomposition and allocation of those functions to different IPS modules. Because NGIPS is intended to employ an Open Architecture, at some point standards will require development to formalize the responsibility of each module in fulfilling the functions. These common functions include:

- Power Management – Normal Conditions
- Power Management – Quality of Service
- Power Management – Survivability
- System Stability
- Fault Response
- Power Quality
- Maintenance Support
- System Grounding

This paper provided alternative implementations for these common functions. Future research and study is needed to determine the optimal solution that should be implemented in the NGIPS open architecture.

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APPENDIX 1 STABILITY OF A SIMPLE DC POWER SYSTEM WITH NONLINEAR LOADS

A simple DC power system with a non-linear load can be modeled as shown in Figure 9.

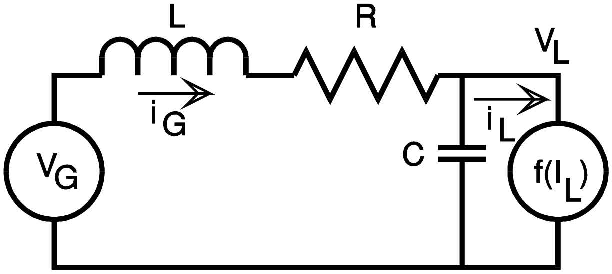

Figure 9: Simple DC Power System with Nonlinear Load

The equations describing this system are:

$$i_G = i_L + C\frac{df(i_L)}{dt} \qquad [1.]$$

$$V_G = f(i_L) + i_G R + L\frac{di_G}{dt} \qquad [2.]$$

Differentiating [1.]

$$\frac{di_G}{dt} = \frac{di_L}{dt} + C\frac{d^2 f(i_L)}{dt^2} \qquad [3.]$$

Now substituting in [2.]

$$V_G = f(i_L) + \left(i_L + C\frac{df(i_L)}{dt}\right)R + L\left(\frac{di_L}{dt} + C\frac{d^2 f(i_L)}{dt^2}\right) \qquad [4.]$$

Expand the derivatives of $f(i_L)$

$$\frac{df(i_L)}{dt} = \frac{df(i_L)}{di_L}\frac{di_L}{dt} \qquad [5.]$$

$$\frac{d^2 f(i_L)}{dt^2} = \frac{df(i_L)}{di_L}\frac{d^2 i_L}{dt^2} + \frac{d^2 f(i_L)}{di_L^2}\left(\frac{di_L}{dt}\right)^2 \qquad [6.]$$

Now substitute in [4.]

$$V_G = f(i_L) + \left(i_L + C\frac{df(i_L)}{di_L}\frac{di_L}{dt} \right) R + L\left(\frac{di_L}{dt} + C\left(\frac{df(i_L)}{di_L}\frac{d^2 i_L}{dt^2} + \frac{d^2 f(i_L)}{di_L^2}\left(\frac{di_L}{dt} \right)^2 \right) \right) \quad [7.]$$

The steady state solution is obtained by setting time derivatives to zero

$$V_G = f(i_{L0}) + i_{L0}R \quad [8.]$$

Use Taylor Series Expansion

$$i_L = i_{L0} + i \quad [9.]$$

$$f(i_L) = f(i_{L0}) + \frac{df(i_{L0})}{di_{L0}} i \quad [10.]$$

Substitute into [7.]

$$0 = \frac{df(i_{L0})}{di_{L0}} i + \left(i + C\frac{df(i_{L0})}{di_{L0}}\frac{di}{dt} \right) R + L\left(\frac{di}{dt} + C\left(\frac{df(i_{L0})}{di_{L0}}\frac{d^2 i}{dt^2} + \frac{d^2 f(i_{L0})}{di_{L0}^2}\left(\frac{di}{dt} \right)^2 \right) \right) \quad [11.]$$

Rearranging

$$0 = \left(\frac{df(i_{L0})}{di_{L0}} + R \right) i + \left(RC\frac{df(i_{L0})}{di_{L0}} + L \right) \frac{di}{dt} + \left(LC\frac{df(i_{L0})}{di_{L0}} \right) \frac{d^2 i}{dt^2} + \left(LC\frac{d^2 f(i_{L0})}{di_{L0}^2} \right) \left(\frac{di}{dt} \right)^2 \quad [12.]$$

Assuming the last element is not significant, the resulting linear differential equation is

$$0 = \left(\frac{df(i_{L0})}{di_{L0}} + R \right) i + \left(RC\frac{df(i_{L0})}{di_{L0}} + L \right) \frac{di}{dt} + \left(LC\frac{df(i_{L0})}{di_{L0}} \right) \frac{d^2 i}{dt^2} \quad [13.]$$

For this system to be stable, all the coefficients have to have the same sign. If the load has a positive incremental resistance $\frac{df(i_{L0})}{di_{L0}}$, then all of the terms will be positive and the system will be stable. If the load has a negative incremental resistance, then since the last term has a negative sign, the conditions for stability are:

$$\frac{df(i_{L0})}{di_{L0}} + R < 0 \quad [14.]$$

$$RC\frac{df(i_{L0})}{di_{L0}} + L < 0 \quad [15.]$$

Now apply this analysis to a constant power load where

$$f(i_L) = \frac{P}{i_L} \qquad [16.]$$

The derivatives of $f(i_L)$ are

$$\frac{df(i_L)}{di_L} = -\frac{P}{i_L^2} = -\frac{V_L^2}{P} \qquad [17.]$$

$$\frac{d^2 f(i_L)}{di_L^2} = 2\frac{P}{i_L^3} = 2\frac{V_L^3}{P^2} \qquad [18.]$$

The conditions for stability are now:

$$-\frac{P}{i_{L0}^2} + R < 0 \quad \text{or} \quad -\frac{V_{L0}^2}{P} + R < 0 \qquad [19.]$$

$$-RC\frac{P}{i_{L0}^2} + L < 0 \quad \text{or} \quad -RC\frac{V_{L0}^2}{P} + L < 0 \qquad [20.]$$

Examining these two conditions, it's clear that stability is harder to achieve as the power P increases. If V_{L0} is considered a constant, for [19.], the source resistance must be reduced as P is increased. From [20], the capacitance must be increased and inductance decreased. Thus for a given system, there is a maximum P for which stability can be assured.

Conditions [19.] and [20.] must be used with caution. Direct simulation of this system has shown unstable behavior when the conditions are negative, but one approaches zero. This unstable behavior is likely due to the ignored nonlinear terms.

From a system perspective, increasing the system voltage V_{L0} is stabilizing for both [19.] and [20.].

Now let's go back to the last term in [12] which we previously ignored …

$$\left(LC\frac{d^2 f(i_{L0})}{di_{L0}^2}\right)\left(\frac{di}{dt}\right)^2 \qquad [21.]$$

$$\left(2LC\frac{P}{i_{L0}^3}\right)\left(\frac{di}{dt}\right)^2 \quad \text{or} \quad \left(2LC\frac{V_{L0}^3}{P^2}\right)\left(\frac{di}{dt}\right)^2 \qquad [22.]$$

If once again we consider V_{L0} a constant, then as P increases, this term vanishes and our assumption that it could be ignored is valid. However, at very low power levels, this term can become very large. The issue now is determining whether this term at low power is "stabilizing" or "destabilizing." First, rearrange equation [12.] and make the substitutions from [17.] and [18.]

$$0=\left(-\frac{V_{L0}^2}{P}+R\right)i+\left(-RC\frac{V_{L0}^2}{P}+L+2LC\frac{V_{L0}^3}{P^2}\left(\frac{di}{dt}\right)\right)\frac{di}{dt}+\left(-LC\frac{V_{L0}^2}{P}\right)\frac{d^2i}{dt^2} \qquad [23.]$$

Multiply equation [23.] by P^2 results in

$$0=\left(-V_{L0}^2P+RP^2\right)i+\left(-RCPV_{L0}^2+LP^2+2LCV_{L0}^3\left(\frac{di}{dt}\right)\right)\frac{di}{dt}+\left(-LCV_{L0}^2P\right)\frac{d^2i}{dt^2} \qquad [24.]$$

In the limit that $P\rightarrow 0$, [24] reduces to

$$0=2LCV_{L0}^3\left(\frac{di}{dt}\right)^2 \qquad [25.]$$

Equation [25.] has a root at the origin and therefore is neutrally stable.

For small values of P, the second term (coefficient of $\frac{di}{dt}$) in [24.] is "stabilized" when $\frac{di}{dt}$ is negative and "destabilized" when $\frac{di}{dt}$ is positive. In reality, because [24.] is a nonlinear differential equation, the terms "stabilizing" and "destabilizing" have very murky definitions. In the context used above, those changes that would result in a coefficient of [24.] becoming more positive are considered "destabilizing" while those that cause a coefficient becoming more negative are considered "stabilizing." From [19.] and [20.], these nonlinear effects are likely not significant in most cases when compared to the stabilizing impact of a small P. At low power levels however, dynamics not modeled here may actually predominate. Thus for this system with a very low P, the dynamic performance is hard to characterize.

The conclusions from this analysis are that for DC power systems with constant power loads, stability is an issue at high power levels (but less so if the system voltage is increased), and non-linear behavior that can not be adequately modeled by linear theory and unmodeled dynamics may become an issue at low power levels.