Optimal Generator Set Loading for Energy Efficiency

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Abstract

Shipboard power systems have typically shared electrical power equally among paralleled generator sets. While this strategy provides system robustness with respect to being able to handle sudden, large changes in load, it is not necessarily the most fuel efficient way to operate the generator sets. With modern advanced controls, possibly augmented with energy storage, generator set loading can now be based on optimal fuel efficiency while still providing system robustness. This paper presents an analytic approach to determining the optimal loading, with respect to fuel economy, of a set of paralleled generator sets for a given total load.

Introduction

Historically, paralleled a.c. generator sets, comprised of a prime mover and generator, have shared loads equally based on the fraction of their rated load. This was accomplished through either frequency droop on the prime mover speed governors, or through cross-compensation signals to the feedback signal on the prime mover speed governors. This method of sharing loads is very robust in that it can respond quickly to changes in loads over the entire combined rated power of the paralleled generator sets. This capability however, comes at the expense of fuel economy. With modern controls, particularly when paired with energy storage, operating generators at the optimal loading can improve fuel economy without sacrificing operational robustness. The question is then, for a given total load, what is the optimal apportionment of load among multiple generator sets?

The fuel consumption of a generator set is usually specified in terms of the specific fuel consumption (sfc) with units of kg/kW-h as a function of the fraction of rated power. This relationship is generally provided in the form of a table or graph. If the generator output is intended to be rectified, then an sfc map in the form of iso-sfc contours plotted on a graph with the x-axis corresponding to the rotational speed of the prime mover or frequency of the generator set, and the y-axis corresponding to the delivered power is preferred (Figure 1). Alternately, a correction factor for variable speed operation of a diesel engine as compared to constant speed operation is provided by Holmefjord et al. (2020) The minimum sfc for a given power is used and the corresponding speed translates into the frequency of power generated. This frequency need not be 60 Hz since it will be immediately rectified. In some cases the sfc data is provided for only the prime mover and must be adjusted to account for the efficiency of the generator.

Multiplying the sfc by the power (kW) results in a fuel rate (kg/h). The objective of optimal generator set loading is minimizing the overall fuel rate (the sum of all the fuel rates of all the online generator sets) for a given total power P while, if possible, operating each generator set within the range bounded by its light load limit and heavy load limit.

Operating a generator set below its light load limit, while possible, will result in increased maintenance costs. Operating a generator set above its heavy load limit, while possible, may result in overloading due to normal fluctuation of load and transient loads. The light load limit is typically about 15% of the rated power (although it may be as large as 40%) and the heavy load limit is typically about 90% of the rated power (although it may as large as 100% if sufficient energy storage and appropriate controls are available to prevent overloading).

A numerical approach to finding the optimum loading for two generator sets providing a total power of *P* with minimal total fuel rate consists of the following steps:

- Estimate the fuel rate at a number of powers for generator set 1 between its light load limit and heavy load limit
- Estimate the fuel rate at a number of powers for generator set 2 where the powers are calculated by subtracting the powers of generator set 1 from *P*.
- Add the corresponding fuel rates for the two generator sets to develop a curve of the total fuel rate as a function of generator loading for total power *P*.
- Find the minimum total fuel rate of this curve which corresponds to the optimal loading of the two generator sets for total power *P*.

Because the powers used in this process are likely not at the powers for which an sfc was provided for each generator set, a means for interpolating between the provided points is required. Particularly at low powers, where the sfc changes rapidly, the method of interpolation can impact the calculated optimum. Interpolating on the fuel rate, rather than sfc will likely result in a better outcome. In any case, understanding the potential errors introduced by the interpolation method is important.

Extending the optimization method described above for two generator sets to three or more generator sets could quickly become cumbersome and require considerable computational time. Solving the optimization problem is more tractable if one recognizes that the addition of a 3rd generator set does not change the optimal relationship of the first two generator sets. Thus, if one creates a fuel rate table for the optimal combination of the first two generator sets, independent of the third (or additional) generator sets, then this fuel rate table along with the fuel rate table for the third generator set can be used in the optimization process used for two generator sets. This process can be extended to an arbitrary number of generator sets.

While the numerical method described above will find an optimum within the limits of accuracy of the interpolation method and number of power increments employed, it can result in loading profiles that are hard to explain. This paper explores analytic solutions to the optimization problem. In particular it explores using a fuel rate interpolation method that fits the fuel rate vs power curve using a polynomial. Applying calculus to these equations enables one to relate the location of minimums and maximums to the polynomial coefficients. This analytic approach, while still subject to the limitations of its interpolation scheme, can be used as a check on the numerical method-based optimization or used directly to determine appropriate generator loading.

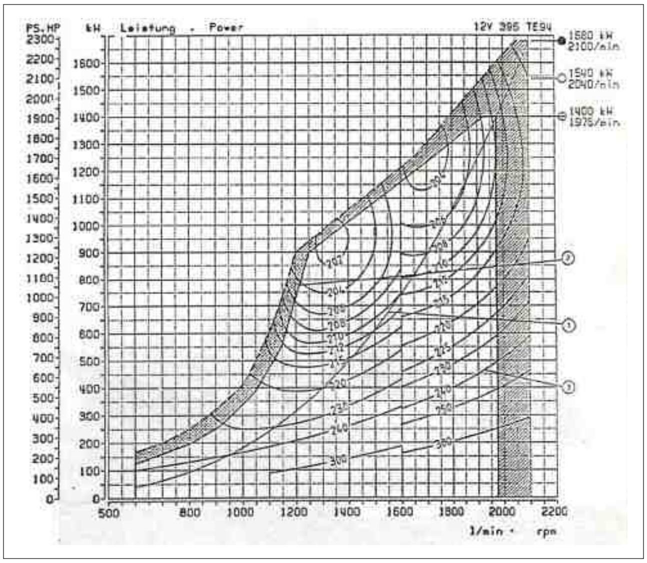

FIGURE 1. Diesel sfc map (Guenther 1989)

Two Generator Sets Using Linear Fuel Rate

If the fuel rate for generator sets is assumed to be linear, then the fuel rate for each generator set can be expressed as:

$$r_{fuelx} = r_{1x} p_x + r_{0x}$$

Where

r_{fuelx}	Fuel rate for generator set *x*
r_{1x}	linear coefficient for fuel rate for generator set *x*
r_{0x}	No load fuel rate for generator set *x*
p_x	Power output for generator set *x*

If we have two generator sets providing power *P*, then the total fuel rate *R* is

$$R = r_{fuel1} + r_{fuel2} = r_{11} p_1 + r_{12}\,(P - p_1) + r_{01} + r_{02}$$

This has a minimum when the derivative of *R* with respect to p_1 is zero

$$\frac{dR}{dp_1} = r_{11} - r_{12}$$

Since the derivative is constant, the minimum occurs at one of the boundaries depending on the sign of the derivative. The boundaries for generator set 1 are based on both generator sets not exceeding their heavy load limit and also not being below their light load light. The lower boundary for generator set 1 is equal to the larger of the light load limit for generator set 1 and *P* minus the heavy load limit for generator set 2. The upper boundary for generator set 1 is the smaller of the heavy load limit of generator set 1 and *P* minus the light load limit of generator set 2. If the derivative is positive, the minimum fuel rate occurs at the lower boundary for generator set 1. If the derivative is negative, the minimum fuel rate occurs at the upper boundary for generator set 1. The power on generator set 2 is equal to *P* minus the power on generator set 1.

This means that for two generator sets operating above the sum of their light load limits, the generator set with the higher r_{1x} should be operated at its light load limit and the generator set with the lower r_{1x} should take up the remaining load until it reaches its rated load. If the generator set with the lower r_{1x} is operating at its heavy load limit, the other generator set should take up the remaining load. If *P* is less than the sum of the light load limits or more than the sum of the heavy load limits, then *P* should be apportioned to the two generator sets such that each is loaded to the same fraction of rated power.

If the two generator sets are identical, or have identical r_{1x}, then any partitioning of *P* among the two generator sets will have the same fuel rate assuming a linear fuel rate. In this case, sharing the load equally is reasonable.

The optimal loading does not depend on the no load fuel rate of either generator set. These no load fuel rates are the "cost of entry" associated with having the two particular generator sets online.

Optimal Fuel Rate With Two Generator Sets

If we assume the fuel rate that can be represented by a cubic equation, the fuel rate for each generator set can be expressed as:

$$r_{fuelx} = r_{3x}{p_x}^3 + r_{2x}{p_x}^2 + r_{1x}p_x + r_{0x}$$

If we have two generator sets providing power *P*, then the total combined fuel rate *R* is

$$R = r_{31}{p_1}^3 + r_{32}(P - p_1)^3 + r_{21}{p_1}^2 + r_{22}(P - p_1)^2 + r_{11}p_1 + r_{12}(P - p_1) + r_{01} + r_{02}$$

Rearranging terms:

$$R = (r_{31} - r_{32}){p_1}^3 + (r_{32}3P + r_{21} + r_{22}){p_1}^2 + (-3P^2r_{32} - 2Pr_{22} + r_{11} - r_{12})p_1 + r_{32}P^3 + r_{22}P^2 + r_{12}P + r_{01} + r_{02}$$

The derivative is given by:

$$\frac{dR}{dp_1} = 3(r_{31} - r_{32}){p_1}^2 + 2(r_{32}3P + r_{21} + r_{22})p_1 + (-3P^2r_{32} - 2Pr_{22} + r_{11} - r_{12})$$

Setting this derivative to 0 and solving for p_1 will find a minimum, maximum, or inflection point. This equation can be solved using the quadratic equation to determine p_1; p_2 can be calculated by subtracting p_1 from *P*. If these two solutions to the quadratic equation are complex, or outside the boundaries of generator set 1, they are discarded from further consideration. Otherwise, the total fuel rate for the set of generators is calculated at these values and at the lower and upper boundaries of generator set 1. The load on generator set 1 corresponding to the lowest combined fuel rate is used.

If the generator sets are identical then

$$R = (3Pr_{31} + 2r_{21}){p_1}^2 - (3P^2r_{31} + 2Pr_{21})p_1 + r_{31}P^3 + r_{21}P^2 + r_{11}P + 2r_{01}$$

$$\frac{dR}{dp_1} = 2(3Pr_{31} + 2r_{21})p_1 - (3P^2r_{31} + 2Pr_{21})$$

Setting the derivative to zero results in:

$$p_1 = \frac{P}{2}$$

Thus, equal sharing is either a minimum or a maximum. The curvature is given by

$$\frac{d^2R}{dp_1{}^2} = 2(3Pr_{31} + 2r_{21})$$

This is positive (indicating a minimum) when

$$P > -\frac{2r_{21}}{3r_{31}}$$

If P equals the expression to the right of the inequality, then equal sharing may or may not be a minimum and must be tested manually. Otherwise the minimum occurs at one of the limits.

Alternately, the first derivative can be calculated at the boundaries for generator set 1. If the derivative is negative at the lower boundary and positive at the upper boundary, then equal sharing results in a minimum combined fuel rate. If the derivative is positive at the lower boundary and negative at the upper boundary, equal sharing results in a maximum combined fuel rate. In this case the minimum combined fuel rate will occur at one of the boundaries for generator set 1. If the derivatives at boundaries are of the same sign, the minimum combined fuel rate will also occur at one of the boundaries for generator set 1.

Optimal Rate With Three Identical Generator Sets

In the case of three identical generator sets, the total combined fuel rate is given by:

$$R = r_3(p_1{}^3 + p_2{}^3 + p_3{}^3) + r_2(p_1{}^2 + p_2{}^2 + p_3{}^2) + r_1(p_1 + p_2 + p_3) + 3r_0$$

$$P = p_1 + p_2 + p_3$$

We can eliminate one of the variables through substitution

$$R = r_3(p_1{}^3 + p_2{}^3 + (P - p_1 - p_2)^3) + r_2(p_1{}^2 + p_2{}^2 + (P - p_1 - p_2)^2) + r_1P + 3r_0$$

A minimum, maximum, or saddle point is located at the point where the partial derivatives are both equal to zero. The partial derivatives are given by:

$$\frac{\partial R}{\partial p_1} = r_3(3p_1{}^2 - 3(P - p_1 - p_2)^2) + r_2\big(2p_1 - 2(P - p_1 - p_2)\big)$$

$$\frac{\partial R}{\partial p_2} = r_3(3p_2{}^2 - 3(P - p_1 - p_2)^2) + r_2\big(2p_2 - 2(P - p_1 - p_2)\big)$$

Setting the partial derivatives to zero and solving for p_1 and p_2 results in the following three possible points for minimums:

$$p_1 = P + \frac{2r_2}{3r_3} \quad p_2 = P + \frac{2r_2}{3r_3} \quad p_3 = -P - \frac{4r_2}{3r_3}$$

$$p_1 = P + \frac{2r_2}{3r_3} \quad p_2 = -P - \frac{4r_2}{3r_3} \quad p_3 = P + \frac{2r_2}{3r_3}$$

$$p_1 = \frac{P}{3} \quad p_2 = \frac{P}{3} \quad p_3 = \frac{P}{3}$$

The first two points are effectively equivalent, with the roles of generator sets 2 and 3 interchanged. These points may not be feasible because one of the power levels may not be in the boundaries for the generator sets. The last point, corresponding to equal loading, should always be within the boundaries for the generators, assuming a sufficiently large P. However, this point may represent a maximum.

The second partial derivatives are given by

$$\frac{\partial^2 R}{\partial {p_1}^2} = 6r_3(P - p_2) + 4r_2$$
$$\frac{\partial^2 R}{\partial {p_2}^2} = 6r_3(P - p_1) + 4r_2$$

If both of these second partial derivatives are positive, the point is a minimum.

For the first two points, the second partial of the fuel rate with respect to p_2 is identically zero, which means the points must be checked manually to determine if they are a minimum or maximum.

$$\frac{\partial^2 R}{\partial {p_2}^2} = 6r_3\left(P - P - \frac{2r_2}{3r_3}\right) + 4r_2 = 0$$

For the third point to be a minimum:

$$\frac{\partial^2 R}{\partial {p_1}^2} = 6r_3\left(P - \frac{P}{3}\right) + 4r_2 > 0$$
$$P > -\frac{r_2}{r_3}$$

If $P = -\frac{r_2}{r_3}$, then the third point must be checked manually.

If the third point cannot be confirmed to be a minimum, then in addition to the points that must be checked manually, the fuel rate should be calculated at the following points:

- generator set 1 loading at the lower boundary and the remaining generator set loading determined by optimizing for 2 identical generator sets.
- generator set 1 loading at the upper boundary and the remaining generator set loading determined by optimizing for 2 identical generator sets.

In the case of three generator sets, a generator set's lower boundary is the greater of the light load limit of the generator set and *P* minus the sum of the heavy load limits of the remaining two generator sets. The upper boundary is the lesser of the heavy load limit of the generator set and *P*, minus the sum of the light load limits of the remaining two generator sets.

The lowest fuel rate among these points is the optimal fuel rate.

Optimal Fuel Rate With 3 Generator Sets, 2 Identical

In the case of three generator sets, two of which are identical, the total combined fuel rate is given by:

$$R = r_{31}{p_1}^3 + r_{32}({p_2}^3 + {p_3}^3) + r_{21}{p_1}^2 + r_{22}({p_2}^2 + {p_3}^2) + r_{11}p_1 + r_{12}(p_2 + p_3) + r_{01} + 2r_{02}$$
$$P = p_1 + p_2 + p_3$$

As before, eliminate p_3 through substitution and calculate the partial derivative with respect to p_1 and p_2 and set to 0:

$$0 = -3r_{32}{p_2}^2 - 6r_{32}p_1p_2 + 6Pr_{32}p_2 - 3r_{32}{p_1}^2 + 6Pr_{32}p_1 - 3P^2r_{32} + 3r_{31}{p_1}^2 + 2r_{22}p_2 + 2r_{22}p_1 - 2Pr_{22} + 2r_{21}p_1 - r_{12} + r_{11}$$

$$0 = -6r_{32}p_1p_2 + 6Pr_{32}p_2 - 3r_{32}{p_1}^2 + 6Pr_{32}p_1 - 3P^2r_{32} + 4r_{22}p_2 + 2r_{22}p_1 - 2Pr_{22}$$

These two equations have 3 sets of possible minimum points:

$$a_1 = 9P^2r_{31} + 6Pr_{21} - 3r_{12} + 3r_{11}$$
$$a_2 = 12Pr_{22}r_{31} + {r_{22}}^2 + 4r_{21}r_{22}$$
$$a_3 = 4{r_{22}}^2r_{31}r_{32}$$
$$a_4 = 12r_{31}(Pr_{22} + r_{12} - r_{11}) + {r_{22}}^2 + 4r_{21}r_{22} + 4{r_{21}}^2$$
$$a_5 = 3Pr_{32} + 2r_{22} + 4r_{21}$$
$$a_6 = 6Pr_{31} + r_{22} + 2r_{21}$$

$$p_1 = P + \frac{2r_{22}}{3r_{32}} \quad p_2 = -\frac{r_{22}}{3r_{32}} - \frac{\sqrt{a_1 r_{32}{}^3 + a_2 r_{32}{}^2 + a_3}}{3r_{32}{}^2} \quad p_3 = -\frac{r_{22}}{3r_{32}} + \frac{\sqrt{a_1 r_{32}{}^3 + a_2 r_{32}{}^2 + a_3}}{3r_{32}{}^2}$$

$$p_1 = \frac{a_5 - 2\sqrt{a_1 r_{32} + a_4}}{3r_{32} - 12r_{31}} \quad p_2 = \frac{-a_6 + \sqrt{a_1 r_{32} + a_4}}{3r_{32} - 12r_{31}} \quad p_3 = P - p_1 - p_2$$

$$p_1 = \frac{a_5 + 2\sqrt{a_1 r_{32} + a_4}}{3r_{32} - 12r_{31}} \quad p_2 = \frac{-a_6 - \sqrt{a_1 r_{32} + a_4}}{3r_{32} - 12r_{31}} \quad p_3 = P - p_1 - p_2$$

The above points should be calculated to determine if they fall within the bounds of each generator set. The fuel rate should be calculated for those points in addition to the ones listed below:

- generator set 1 loading at the upper boundary and the remaining generator set loading determined by optimizing for 2 identical generator sets for *P* minus the upper boundary of generator 1.
- generator set 1 loading at the lower boundary and the remaining generator set loading determined by optimizing for 2 identical generator sets for *P* minus the lower boundary of generator set 1.
- generator set 2 loading at the upper boundary and the remaining generator set loading determined by optimizing for 2 different generator sets for *P* minus the upper boundary of generator set 2.
- generator set 2 loading at the lower boundary and the remaining generator set loading determined by optimizing for 2 different generator sets for *P* minus the lower boundary of generator set 2.

A generator set's lower boundary is the greater of the light load limit of the generator set and *P*, minus the sum of the heavy load limits of the other generator sets. The upper boundary is the lesser of the heavy load limit of the generator set and *P* minus the sum of the light load limits of the other generator sets.

The lowest fuel rate among these points is the optimal fuel rate.

Optimal Fuel Rate With Four Identical Generator Sets

In the case of four generator sets, all of which are identical, the total combined fuel rate is given by:

$$R = r_3(p_1{}^3 + p_2{}^3 + p_3{}^3 + p_4{}^3) + r_2(p_1{}^2 + p_2{}^2 + p_3{}^2 + p_4{}^2) + r_1(p_1 + p_2 + p_3 + p_4) + 4r_0$$

$$P = p_1 + p_2 + p_3 + p_4$$

As before, eliminate p_4 through substitution and calculate the partial derivative with respect to p_1, p_2 and p_3, set to 0, and solve for p_1 through p_4 results in the following potential minimum points:

$$p_1 = \frac{P}{2} + \frac{r_2}{3r_3} \quad p_2 = \frac{P}{2} + \frac{r_2}{3r_3} \quad p_3 = \frac{P}{2} + \frac{r_2}{3r_3} \quad p_4 = -\frac{P}{2} - \frac{r_2}{r_3}$$

$$p_1 = \frac{P}{4} \quad p_2 = \frac{P}{4} \quad p_3 = \frac{P}{4} \quad p_4 = \frac{P}{4}$$

The above points should be calculated to determine if they fall within the bounds of each generator set. The fuel rate should be calculated for those points in addition to the ones listed below for each point, where all the generator set loads are between the light load and heavy load values:

- generator set 1 at the upper boundary and the remaining generator set loading chosen from optimizing for 3 identical generators for power equal to *P* minus the heavy load limit.
- generator set 1 at the lower boundary and the remaining generator set loading chosen from optimizing for 3 identical generators for power equal to *P* minus the light load limit

A generator set's lower boundary is the greater of the light load limit of the generator set and *P*, minus the sum of the heavy load limits of the other generator sets. The upper boundary is the lesser of the heavy load limit of the generator set and *P*, minus the sum of the light load limits of the other generator sets.

The lowest fuel rate among the points that fall within the boundaries is the optimal fuel rate.

Optimal Fuel Rate With An Arbitrary Number Of Generator Sets

Analytical solutions for combinations of up to three generator sets have been presented in the previous sections. For combination of four to nine generator sets, the optimal fuel rate can be determined by partitioning the generator sets into two or three composite groups of three or less generator sets. For each composite group, the optimal fuel rate is determined for a range of power levels (typically 10 or more over the range from the light load limit to the heavy load limit). A cubic curve is fit to this data to determine the fuel rate coefficients for the composite group. In determining the optimal fuel rate, these composite groups are treated as a single generator set when applying the methods of the previous sections.

For example, the optimal fuel rate for four generator sets comprised of two generator sets of type "A" and two generator sets of type "B" can be calculated by first creating a composite group consisting of one generator set of type "A" and another of type "B", developing the cubic coefficients for this composite group, then using these coefficients to calculate the optimal fuel rate using the method for two identical generator sets. The method for first combining generator sets of type "A" and "B" and then combining two composite groups of type "AB" are described above in the section titled "Optimal Fuel Rate With Two Generator Sets."

Errors may be introduced when developing the coefficients for the composite groups. For this reason, once the load for each generator set has been determined, the fuel rate for each generator set should be directly calculated from the original set of coefficients. The total fuel rate can then be calculated by summing up the fuel rates for the individual generator sets.

Example

Consider a ship power system configuration with two gas turbine and two diesel engine generator sets where the generator sets can all be paralleled. Table 1 presents representative sfc data for the diesel (from Skjong et al. 2017) and gas turbine (from USNA 1979) generator sets. Light load for both types of generator sets is assumed to be 15% or less of the rated power. Heavy load for both generator sets is assumed to be between 90% and 100% of rated power. Converting the sfc data to a fuel rate, then curve fitting to a third order polynomial results in

Gas Turbine		Diesel	
Rating (kW)	18925	Rating (kW)	5000
x	sfc (kg/kW-h)	x	sfc (kg/kW-h)
0.224	0.427	0.100	0.300
0.299	0.366	0.200	0.260
0.358	0.335	0.300	0.241
0.463	0.305	0.400	0.226
0.597	0.274	0.500	0.217
0.694	0.262	0.600	0.210
0.746	0.256	0.700	0.205
0.821	0.250	0.800	0.200
1.000	0.244	0.900	0.199
		1.000	0.198

TABLE 1. Prime mover sfc data

		Rated Power (kW)	r3	r2	r1	r0
gas turbine	G	18925	2.298E-10	-6.949E-06	0.2477	857.9
diesel	D	5000	2.727E-09	-2.567E-05	0.2522	30.4

TABLE 2. Prime mover fuel rate coefficients

Power (kW)	Optimal Fuel Rate (kg/h)	G1 (kW)	G2 (kW)	D1 (kW)	D2 (kW)
Two Gas turbine generator sets (G1 and G2)					
6000	3089	2839	3161		
7000	3295	2839	4161		
8000	3493	2839	5161		
10000	3870	2839	7161		
12000	4231	2839	9161		
16000	4949	2839	13161		
20000	5735	2968	17033		
24000	6454	12000	12000		
28000	7190	14000	14000		
32000	7968	16000	16000		
Two diesel generator sets (D1 and D2)					
2000	517			750	1250
4000	879			750	3250
5000	1054			750	4250
6000	1254			1500	4500
7000	1431			3500	3500
8000	1606			4000	4000

TABLE 3. Optimal loading for combinations DD and GG

the coefficients listed in Table 2. The problem is to identify the combination of at least two generator sets (and their allocation of load) that result in the lowest fuel consumption for a variety of total loads.

If we consider the gas turbine generator set to be of type "G" and the diesel generator set to be of type "D", the following

Power (kW)	Optimal Fuel Rate (kg/h)	G1 (kW)	G2 (kW)	D1 (kW)	D2 (kW)
1 Gas Turbine and 1 Diesel Generator Sets (G1 and D1)					
4000	1803	2839		1161	
5000	1993	2839		2161	
6000	2168	2839		3161	
7000	2342	2839		4161	
8000	2544	3500		4500	
10000	2942	5500		4500	
12000	3299	11250		750	
16000	4031	11500		4500	
20000	4778	15500		4500	
1 Gas Turbine and 2 Diesel Generator Sets (G1, D1, and D2)					
5000	2059	2839		750	1411
6000	2244	2839		750	2411
7000	2417	2839		750	3411
8000	2594	2839		750	4411
10000	2969	2839		3581	3581
12000	3332	3000		4500	4500
1600	4102	14500		750	750
20000	4836	11000		4500	4500
24000	5574	15000		4500	4500
2 Gas Turbines and 1 Diesel Generator Set (G1, G2, and D1)					
7000	3346	2839	2839	1323	
8000	3532	2839	2839	2323	
10000	3882	2839	2839	4323	
12000	4289	2839	4661	4500	
16000	5018	2839	12411	750	
20000	5751	2839	12661	4500	
24000	6524	2839	16661	4500	
28000	7255	13625	13625	750	
32000	7990	13750	13750	4500	
36000	8761	15750	15750	4500	

TABLE 4. Optimal loading for combinations GD, GDD and GGD

six combinations of generator sets are possible: DD, GD, GG, GDD, GGD, and GGDD.

Two identical generator sets: combinations DD and GG

As described earlier, the optimal solution is for equal sharing if the following condition holds true:

$$P > -\frac{2r_2}{3r_3}$$

If it does not hold true, then the optimal solution will occur when one of the generator sets is operating at either its light load condition or at its heavy load condition.

For the gas turbine generator set, equal sharing occurs for a total load greater than about 20,200 kW. For the diesel generator set, equal sharing occurs for a total load greater than about 6,270 kW. The optimal loading and optimal fuel rate for the applicable total powers of interest are shown in Table 3.

GD Composite Fuel Rate	
x (Fraction of Rated Power)	**Optimal Fuel Rate (kg/h)**
0.1	1454
0.2	1954
0.3	2374
0.4	2859
0.5	3292
0.6	3727
0.7	4166
0.8	4610
0.9	5091
1	5606

GD Composite Fuel Rate Coefficients					
	Rated Power (kW)	**r3**	**r2**	**r1**	**r0**
GD	23925	1.123E-10	-4.299E-06	0.2353	898

TABLE 5. Composite Generator Set for 1 Gas Turbine and 1 Diesel Generator Sets

Two and three generator sets: combinations GD, GDD, and GGD

The procedures in sections 3 and 5 can be used to determine the optimal fuel rate and generator loading for the combinations GD, GDD, and GGD. These procedures identify a number of candidate generator loading sets that may prove to have the minimum fuel rate, calculate the fuel rate at these generator loading sets, and use the set with the minimum fuel rate. The results of these calculations are shown in Table 4.

Four generator sets: combination GGDD

Calculating the optimal loading for all four generator sets requires the creation of a composite generator set for the GD combination. Table 5 lists the points used for curve fitting as well as the resulting coefficients from the curve fitting.

The optimal loading for all four generator sets is the optimal loading for two identical GD composite generator sets. The optimal loading for powers above about 25500 kW is for equal sharing among the composite generator sets (the diesels will both operate at the same power, and the gas turbines will both operate at the same power, but the load sharing between diesels and gas turbines may not be shared proportionally by rated power). Below this power, one of the composite generator sets

Power (kW)	Optimal Fuel Rate (kg/h)	G1 (kW)	G2 (kW)	D1 (kW)	D2 (kW)
8000	3601	2839	2839	750	1573
10000	3955	2839	2839	750	3573
12000	4345	2839	3911	750	4500
16000	5089	2839	11661	750	750
20000	5822	2839	11911	750	4500
24000	6577	2839	15911	750	4500
28000	7322	13250	13250	750	750
32000	8062	11500	11500	4500	4500
36000	8790	13500	13500	4500	4500
40000	9556	15500	15500	4500	4500

TABLE 6. Optimal loading for combinations GGDD

	Optimal Fuel Rate (kg/h)					
Power (kW)	DD	GD	GG	GDD	GGD	GGDD
2000	**517**					
4000	**879**	1803				
5000	**1054**	1993		2059		
6000	**1254**	2168	3089	2244		
7000	**1431**	2342	3295	2417	3346	
8000	**1606**	2544	3493	2594	3532	3601
10000		**2942**	3870	2969	3882	3955
12000		**3299**	4231	3332	4289	4345
16000		**4031**	4949	4102	5018	5089
20000		**4778**	5735	4836	5751	5822
24000			6454	**5574**	6524	6577
28000			**7190**		7255	7322
32000			**7968**		7990	8062
36000			8812		**8761**	8790
40000						**9556**

TABLE 7. Comparison of generator set combinations

will operate at its heavy load or light load condition. Once the loading on the composite generator sets has been determined, the optimization routine for GD can be used for each of the composite loadings to determine the actual loading of each generator set. Table 6 depicts the results of this algorithm.

Comparison

Table 7 summarizes the optimal fuel rates for each of the generator set combinations for the each of the total ship power levels of interest. The minimum fuel rate for a given power is bolded. With the exception of a total power of 36,000 kW, the results are expected: minimize total number of generator sets online and prioritize diesels over gas turbines. For 36,000 kW however, the most fuel efficient combination is three generator set GGD, which has better efficiency than the two generator set GG. Adding the diesel online reduces the overall fuel rate.

Power (kW)	Config	Minimum Fuel Rate (kg/h)	Equal Loading Fuel Rate (kg/h)	Equal Loading % of Optimal
2000	DD	517	519	100.42%
4000	DD	879	908	103.31%
5000	DD	1054	1086	103.03%
6000	DD	1254	1259	100.40%
7000	DD	1431	1431	100.00%
8000	DD	1606	1606	100.00%
10000	GD	2942	2967	100.83%
12000	GD	3299	3324	100.77%
16000	GD	4031	4035	100.08%
20000	GD	4778	4783	100.10%
24000	GDD	5574	5583	100.17%
28000	GG	7190	7190	100.00%
32000	GG	7968	7968	100.00%
36000	GGD	8761	8766	100.06%
40000	GGDD	9556	9566	100.10%

TABLE 8. Comparison of Optimal Fuel Rates with Equal Sharing Fuel Rates

Table 8 compares the fuel rate of the optimal loading with equal sharing. For many of the power levels, the optimal fuel rate is the same as or only marginally better than the equal sharing fuel rate. At some power levels, particularly when the generator sets are not highly loaded, such as the cases for 4000 kW and 5000 kW, worthwhile savings are possible with the optimal loading.

Impact On Controls

The load assigned to each generator set using the above algorithms is a reference value that the speed governor on each prime mover uses as part of its speed regulation algorithm. How the system behaves to small disturbances in the total load is the subject of small-signal stability, while the system behavior in response to large disturbances is the subject of large-signal stability.

In an a.c. system, load sharing can be implemented through droop control by having each generator set regulate the fuel command so that the power reflects a droop characteristic. A droop characteristic is typically displayed on a graph with power on the x-axis and frequency on the y-axis. The characteristic is typically specified as a line with a no-load frequency and a slope determined by a specified percentage drop in frequency at full load. An alternate approach is isochronous control where each generator set regulates the fuel command based on system frequency and a load sharing signal from a system controller or from the other paralleled generators. This second approach typically results in better frequency regulation of the power system, but requires external signals to remain stable. The droop method does not require external signals to maintain stability, but without the external signals, the frequency regulation will suffer. See Doerry (2020) and Al-Falahi et al. (2018) for more details on droop and isochronous control.

When using droop, a system controller can continuously adjust the droop characteristics at each generator set to implement any desired load sharing among generator sets while maintaining frequency regulation. A vertical droop characteristic is equivalent to commanding a specific power without regard to frequency. A horizontal droop characteristic is equivalent to commanding a specific frequency without regard to power. A generator set with a horizontal droop characteristic is often called a “slack generator” since it will automatically adjust to small changes in total load. Other generator sets paralleled with a slack generator will provide the power on their droop characteristic corresponding to the slack generator’s frequency.

With the assumption that the various speed governors of paralleled generator sets do not interfere with one another, small signal stability quickly focuses on the dynamics of the slack generator; its purpose is to ensure a continuous balance of power generation and load so that the frequency does not increase or decrease from its set point.

If, however, the load abruptly changes (or a generator set trips offline) such that the power provided by online generator sets with assigned power levels is greater than the load, or is less than the load minus the rating of the slack generator, then the slack generator will not be able to balance generation and load; the system may become large signal unstable. If the system control is fast enough to appropriately adjust the set points for every generator, the system may remain stable.

For power levels where equal sharing of two or more generator sets is optimal, it may prove advantageous to operate the sharing generators in droop mode instead of using a slack generator. The system control can continually adjust the no-load frequency setting of the sharing generators to keep the frequency at the operating load near the desired frequency. The two sharing generators will evenly split the load apportioned to them over their combined rating. If there are three or more generators paralleled, the remaining generator can be commanded to provide a fixed power level.

Nonlinear droop characteristics have also been proposed. (e.g. Chen et al. 2019, Gao et al. 2019). The nonlinearities can be used to achieve a number of different goals to include better load sharing at high power levels, better frequency control at low power levels, integration of energy storage, etc.

Energy storage of sufficient energy and power capacity can facilitate optimal generator set loading by enabling the system to better react to sudden changes in load. Energy storage can perform peak shaving when the load temporarily exceeds the capacity of the swing generator, or can provide additional load should the load on the swing generator drop too low. Simulation will likely be required to determine the minimum amount of energy and power capacity required of the energy storage.

The optimal loading of generator sets can change rapidly as the total ship load changes. To limit the impact of this rapid change, consideration should be given to deviating from the optimal profile to reduce the rate of change of the loading on a generator as the total ship load changes. This can be accomplished by modifying the profiles directly, introducing hysteresis, applying a filter to the optimal profile, or a combination of these strategies.

The optimal loading of generators is sensitive to small variations of the sfc curve. In operation it may prove advantageous for the control system to constantly perturb the operating point to see if a nearby operating point is more fuel efficient. This is analogous to “maximum power point tracking” algorithms used to maximize output of solar arrays. (El-Khozondar et al. 2016)

In some cases, it may be possible with a very small increase in fuel consumption to operate fewer generator sets online than indicated by optimal loading. This may be desirable because maintenance costs are typically a function of prime mover operating hours. The small increase in fuel efficiency with optimal loading is offset by higher maintenance costs.

The ability of U.S. naval ships to implement optimal load sharing has recently been advanced through the development of automation for shore power switchboards. As described by Tan, Adams and Pacheco (2019), the new automation system enables a level of automatic generator control not previously implemented on U.S.N ships. In particular, the ability to have generator sets operate as a base load with the real and reactive power specified is a major step towards being able to implement optimal generator loading.

CONCLUSIONS

For the particular example analyzed in this paper, the savings from using optimal loading is likely not worthwhile over most of the total range of loads. However, within specific ranges of total loads, the savings may be significant enough to warrant implementing optimal loading. This is particularly true if the operational profile indicates a significant amount of time will be spent within these specific ranges. The savings could also be significant in unmanned systems where on-station time is highly valued.

When conducting early stage design of shipboard power systems, it may prove useful to employ the analysis method described in this paper to determine if optimal generator set loading is worthwhile. If worthwhile, simulation and testing of the control system approach should be performed to ensure the system is stable and can handle large power transients. NEJ

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