Framework for Analyzing Modular, Adaptable, and Flexible Surface Combatants

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Ongoing cost escalation in excess of general inflation is exerting economic pressure on U.S. naval ship acquisition programs. Concurrently, evolving and unpredictable national security needs are raising the level of uncertainty regarding future ship design requirements. This problem is particularly acute for surface combatants. Modular, adaptable, and flexible designs are becoming naturally attractive; however, flexibility may incur additional cost. This paper presents a framework for determining how much of what type of modularity, adaptability, and flexibility features to incorporate into a surface combatant design to enable the warship to remain operationally relevant over its design service life in an affordable manner. This framework is based on the principles of Real Options Analysis.

KEY WORDS:

Real options analysis; modularity; flexibility; adaptability; warship design; naval force structure.

INTRODUCTION

In a recent assessment of the Navy's long range shipbuilding plan, the Congressional Budget Office observed that since 1985, "...the average difference between the rate of increase in the Navy's shipbuilding cost index and that in the GDP price index has been about 1.3 percentage points per year" (Labs, 2015). Long-run cost growth in excess of inflation in the general economy increases the economic pressure on ship acquisition programs, reinforcing the need to make optimal choices in terms of force architecture, ship designs, and industrial base configuration.

One response to the increasing acquisition cost of a capital asset would be to increase the asset's service life. For similar asset classes, and with other factors held equal, higher initial cost will typically justify a longer economic life as the additional cost of maintenance, repair and modernization will appear attractive when compared to the high cost of acquiring a replacement. If selective flexibility and modularity features were designed in to reduce later expenditures on modernization, without an overbalancing penalty to initial cost, then economic life could be extended further. Changes in ship design requirements or in the relative costs of acquisition versus operating and support could alter the balance. See Table 1 for a few indicative examples of the basic service life dynamics of low, medium, and high acquisition cost merchant ships.

Table 1: Initial cost and service life.

Ship type	Price, \$ millions	Average age at demolition	
VLCC	93.5	24.1 years	
LNG	199	36.8	
Cruise ship	(high)	51.7	

VLCC: Very large crude carrier, 200,000 dwt and above

LNG: Liquified natural gas carrier, c. 160,000m³

Data: Clarksons, 2016; data are averages during 2015; cruise ship pricing is not included.

Merchant trading vessels are designed to requirements derived from corporate business plans (Buetzow and Koenig, 2003). Those plans respond to identified future business opportunities. The ship mission is concretely specified and this creates risk due to the shipowners' imperfect ability to predict the future. Provisions for flexibility to accommodate significant in-service mission changes are not generally included in the ship designs. Mission modernizations and conversions are occasionally done in response to changes in mission and/or operational economics. More drastic mission change is handled through vessel sales and purchases. When that is not enough, the company risks bankruptcy. A cautionary example occurred in 1986, when a leading shipping company. United States Lines, was forced to file for bankruptcy due to having designed its Jumbo Econ ships under a grossly incorrect assessment of future operational requirements (Rasky, 1986).

As with merchant fleets, naval force structures respond to changing future requirements. However, the response mechanisms of navies are conditioned by two essential drivers: the high cost of naval ships (with consequent long planned service life), and the tightly integrated nature of the design of certain naval ship types.

The high cost of naval ships provides a strong incentive to base their planned economics on a long service life. Long life creates a need for mid-life technology or mission refresh. For some ship types such as aircraft carriers this is not an issue as new systems can be readily accommodated. Surface combatants, on the other hand, pose a distinct problem in that their mission systems are tightly integrated into the ship vehicle system. This makes non-modular designs difficult to technologically refresh. In the post-World War II era, the inability to economically respond to evolving requirements and technologies has caused dozens of U.S. Navy surface combatants to be decommissioned years before their planned service life was fulfilled. For example, the average service life of the 31-ship Spruance class was 23.6 years; the four nuclearpowered Virginia class cruisers were in service for only 17.7 years (Koenig et al 2009).

In other cases, time-consuming and costly conversions on surface combatants were needed to keep them operationally viable. Examples include the several-years duration mid-life conversions of the USS *Chicago* (CA 136) and USS *Albany* (CA 123) from heavy cruisers to missile cruisers, necessitated by a drastically changed mission need.

Cost and asset management are not the only drivers, however. Strategic concerns can override, in both the commercial and military environment. For example, a manufacturing company may engage in a strategy of uneconomic, temporary lossmaking pricing ("predatory pricing"), in order to drive a more thinly capitalized competitor out of business. Strategies and counter-strategies are formulated and implemented, in which one of the tantamount objectives is to render the competitor's preconceptions and plans irrelevant. The same motivation is present in the arena of military conflict, which Gray (2005) described as "a race between belligerents to correct the consequences of the mistaken beliefs with which they entered combat". This leads to a need to adapt to ever-changing conditions during peacetime war planning and, especially, during hostilities.

In response to evolving and unpredictable national security needs, how can navies avoid the two unattractive alternatives of (1) early retirement or (2) extremely costly modernization? Stated another way, the problem is to increase the ability of the ship to be quickly and economically reconfigured in the future, either for temporary missions or for a permanent capability change. Designing in modular, adaptable, and flexible features would be an effective answer. There is a problem, however: those design features must be paid for. Bertram (2005) sorted the costs into four categories:

- 1. Higher initial design effort
- 2. Reduced design freedom (possibly retarding technological progress)
- 3. Usually higher weight
- 4. Usually higher space requirement

So, as with so many other issues in ship design, there is a trade space. The problem is to determine how much of what type of modularity, adaptability, and flexibility features to incorporate into a surface combatant design to enable the warship to remain operationally relevant over its design service life. This paper presents a framework for decision making in this area.

In making this determination, the manner in which questions are posed for analysis is very important. This paper proposes that the determination depends on a number of uncertain parameters, and the vector of these uncertain parameters forms an uncertainty space that is a function of time. The uncertainty parameters may include elements such as a future adversary's capability in a warfare area, future technology breakthroughs, or the conflict environment (preparing for major combat operations, major combat currently ongoing, regional conflict, or peacetime). For this paper the uncertainty space is evaluated at discrete time steps (typically annually) and is assumed to be representable by a Markov chain of uncertainty spaces. The configuration of the ship, tactics, force architecture, and the status of R&D projects are viewed as a configuration vector modeled as a time dependent vector represented by a Markov chain (where the time steps are typically the same as that for the uncertainty space). The modernization process and the initial design of the ship are assumed constant and comprise a design vector. To summarize these three parameters:

- 1. Uncertainty space (the operational environment)
- 2. Configuration vector (ship, force structure, tactics, status of R&D projects)
- 3. Design vector (initial design of the ship, modernization process)

This paper posits that two or more alternatives for the design vector are under consideration. Each alternative is evaluated to determine where the evolving configuration vector has unacceptable, acceptable, and superior performance with respect to operational relevance within the uncertainty space as a function of time. If possible, total ownership cost incurred to date (with uncertainty estimates) is evaluated for each alternative across the uncertainty space as a function of time. By comparing the magnitude of operational relevance and cost for different alternatives under different uncertainty space trajectories, value-based decisions can be made. Real options analysis provides the basic construct.

REAL OPTIONS

An option is a contract giving its owner the right, but not the obligation, to buy (call) or sell (put) a security or other financial asset (the underlying asset) at a specified price (the strike price) during a set time horizon or on a specific date (exercise date).

Consider a stock whose current price is \$40 per share. A call option is purchased for \$3 with a strike price of \$45, expiring in two months. Two months later, the stock is worth \$55. The option is exercised, the stock is purchased for \$45, sold for \$55, and the profit is \$7 less the transaction cost of purchasing the option at the outset. On the other hand, if the stock was worth, say, \$40 (unchanged) at the end of the two month life of the \$45 call option, the option expires worthless. The purchaser of the option is out the \$3 cost of the option plus the transaction cost of the purchase. If the value of the stock were to be fixed permanently at \$40, then there would be no option contracts written as they would be pointless (and worthless). Option value depends on future uncertainty.

Although this example was purely financial, the analogy to ship and force structure design is clear. The idea of real options analysis in naval force structure formulation and naval ship design is based on future requirements uncertainty, and recognition that the opportunity to make certain kinds of future decisions on ship design characteristics has value which changes over time, that value must be paid for, and it expires within some future time horizon. Conventional business case analysis methods do not take account of this embedded optiontype value. Decision makers implicitly understand that such value exists even if they cannot describe it quantitatively. So decisions are made based on intuition and judgment. This is a well-known issue in R&D planning (Hounshell, 1998). In real options analysis terms, the initial design within the design vector includes the purchase of options in the design (such as modularity features). The configuration vector represents the cumulative effect of options in the design that have been exercised to date, as well as options on the design; i.e. modifications for which features have not been explicitly provided (Koenig 2009, Page 2011). The modernization process within the design vector defines the work necessary to evaluate the uncertainty space and decide how and when to exercise the options. The payoff of the option is represented by the evaluation of the configuration vector in terms of unacceptable, acceptable, and superior operational performance. Unacceptable operational performance is considered a capability gap.

This concept can be extended to include the entire class of ships (or even the entire fleet) as part of the design vector and the configuration vector.

Doerry (2012) provides a non-exhaustive list of eight modularity and flexibility technologies that can be considered real options for future warships:

- Modular Hull Ship
- Mission Bays
- Container Stacks
- Weapon Modules
- Aperture Stations
- Off-Board Vehicles
- Electronic Modular Enclosures (EME)
- Flexible Infrastructure

These technologies require an upfront investment, but may prove economical over the vessel's service life. Real Option Analysis is a tool for determining how much of which technologies one should invest in to minimize the projected total ownership cost.

IMPACT OF AFFORDABILITY ON REAL OPTIONS ANALYSIS

As detailed by Mun (2006), a traditional real options analysis presumes the following requirements hold:

- a) A financial model must exist
- b) Uncertainties must exist
- c) Uncertainties must affect decisions when leadership is actively managing the project and these uncertainties must affect the results of the financial model
- d) Management must have strategic flexibility or options to make mid-course corrections when actively managing the projects
- e) Management must be smart enough and credible enough to execute the options when it becomes optimal to do so

For the Navy, the financial model (a) needs to account for affordability. Affordability is not exclusively a matter of cost; a reduction in cost does not necessarily cause an increase in affordability. Affordability is the willingness to spend budget authority on a system. How much the government is willing to spend to modernize and upgrade a ship depends on a complex interaction between many factors including the nature and immediacy of the geopolitical threat, prospective employment at defense contractor production facilities versus other local economic opportunities, the prior record of reliability in program cost estimates, a number of other technical and managerial factors, and finally the fiscal environment.

Affordability considerations place a constraint on requirement (d). Management has limited flexibility, and the degree of flexibility in a given year is uncertain. Hence, while a capability gap may present itself to the modernization process, the gap may not be able to be effectively filled because the fiscal environment (limited budget authority) may place the upgrade priority below the cut-line (hence not affordable). In another fiscal environment, the funds would be available to fill the same gap (hence affordable). The uncertainty space could model the fiscal environment based on the defense environment (peace time, regional conflict, preparation for major conflict, in major conflict, etc.) The modernization process would be sensitive to the fiscal environment to determine the magnitude of effort that is capable of being expended each year.

This implies that the lowest total ownership cost (independent of affordability) may not be the best answer ... the ability to rapidly adapt when funds are available may have greater value. The significance of a capability gap also depends on the defense environment; a capability gap in peacetime is less pressing than the same gap during a major conflict.

DESIGN VECTOR

The design vector consists of the initial ship configuration at delivery, initial tactics to employ the ship and the modernization process. One can think of the design vector describing the starting point for the configuration vector, and the rule set for evolving the configuration. The modernization process includes the work to identify potential capability gaps, to prioritize R&D to develop system upgrades to fill the gap, to develop new tactics to either fill the gap directly or in concert with system upgrades, and to actually upgrade the ship configuration. The modernization process itself can be a function of time, but is assumed not to be stochastic; the same uncertainty space trajectory should always result in the same configuration vector trajectory.

The design vector may include modularity, adaptability, and flexibility options that enable more affordable and timely responses to capability gaps once they are identified. Figure 1 is an example of a notional simplified design vector. Actual design vectors are likely to be more complex.

DESIGN VECTOR				
Modular Hull Ship NO 64 cell VLS WM /			WMA	
Mission Bay	NO	32 cell VLS WM B		
Container Stack	NO	5 inch gun WM A		
Weapon Modules A	2	37 mm gun WM C		
Weapon Modules B 1 37 mm gun WM		WM C		
Weapon Modules C	4	SEA-RAM	WM C	
Aperture Station A	3	3 CIWS WMC		
Aperture Station B	2 ATT SWAP-C		SWAP-C	
Boats 2				
Aircraft	2	SPS-64	AS B	
EME	YES	SPS-67	AS B	
Flexible Infrastructure	YES	SPY-1D	AS A x 3	
Removal routes	YES			
Electrical SLA	1 MW	Tactics standard		
Cooling SLA 280 tons				
	800 mt	3 month modernization	every 2	
Weight SLA		availablility	years	
	E	9 month modernizaition	every 6	
KG SLA	.5 meters	availability	years	

Figure 1: Notional Simplified Design Vector

CONFIGURATION VECTOR

The configuration vector describes the evolving ship over time, evolving tactics for its employment, and the state of evolving R&D and system development. This vector is made up of the information in the design vector (Figure 1), plus cost data. The cost data includes the original acquisition cost, the R&D cost for modernization, the costs for evaluating the uncertainty space, and the actual cost of implementing the modernization.

The ship, its tactics, and its associated R&D for modernization are viewed as an adaptive system; the configuration vector describes the state of this system as it adapts to the uncertainty space. This adaptation is a manifestation of options that have been exercised over the life of the ship. The configuration vector captures the implications of the presence or absence of modularity and flexibility features.

Figure 2 illustrates how the configuration vector can evolve differently in response to differences in how the uncertainty space could evolve following different uncertainty space trajectories. The data fields of the configuration vector at delivery and the subsequent configuration vectors are as discussed above. In the figure, changes in the colored lines indicate that a system has been replaced or modified. (The exact nature of those systems is not legibly shown, as it is not important for demonstrating the process flow). When changes are incorporated or what changes are actually installed are not pre-planned, but rather are determined when the uncertainty space indicates it is advantageous to do so. In this manner, the analysis captures management's flexibility in the future to best configure the ship in response to the changing uncertainty space.



Figure 2: Evolving Configuration Vectors for different Uncertainty Space Trajectories.

EVALUATING THE CONFIGURATION VECTOR

The configuration vector is evaluated over time to assess its operational relevance.

For each location in an uncertainty space trajectory, the operational relevance of the configuration vector is assessed where possible on quantitative analysis, but presented in terms of lumped characterizations such as unacceptable, acceptable, and superior performance with respect to operational relevance. The affordability of each configuration vector is assessed in terms of constrained or unconstrained. Constrained implies that more could have been done to correct an unacceptable operational performance, but the fiscal environment prevented sufficient investment. Unconstrained implies that the fiscal environment was not a factor for unacceptable operational performance in the current time increment.

The boundaries for the discretization should be well defined and the sensitivity of the results to these boundaries should be explored.

UNCERTAINTY SPACE

As stated in the introduction the uncertainty space is evaluated at discrete time steps (typically annually). The series of uncertainty spaces over time comprise an uncertainty space trajectory. As depicted in Figure 3, the uncertainty space may include elements such as a potential adversary's capability in a warfare area, potential technology breakthroughs, or whether the Nation is preparing or in major combat operations, in regional conflict, or operating in a peacetime mode. The uncertainty space can be modeled as either adaptive, or nonadaptive. The trajectory of an adaptive uncertainty space is influenced by the configuration vector; it captures a potential opponent's response to the evolving configuration vector. In this manner, Game theory can be incorporated into the framework. A non-adaptive uncertainty space is independent of the configuration vector.

UNCERTAINTY SPACE				
World Conflict State Peace Adversary 1 ASW level 8				
Adversary 1 conflict No Adversary 1 AAW level 7		7		
Adversary 2 conflict No Adversary 1 SW level 7		7		
Adversary 3 conflict No Advers		Adversary 2 ASW level	4	
		Adversary 2 AAW level	5	
Key Technology 1 available	No	Adversary 2 SW level	3	
Key Technology 2 available	No	Adversary 3 ASW level	2	
Key Technology 3 available	No	Adversary 3 AAW level	5	
Key Technology 4 No Adversary 3 S		Adversary 3 SW level	5	

Figure 3: Notional Simplified Uncertainty Space

While a non-adaptive uncertainty space may not perfectly represent reality, it may prove more useful in comparing different system alternatives — both may be exposed to the same uncertainty space trajectory which may provide more insight to decision makers. Even if an adaptive uncertainty space is employed, it may be insightful to apply the uncertainty space trajectory of each alternative to the other alternatives (in these cases as non-adaptive uncertainty spaces). Hence the modeling environment should be able to capture both adaptive and non-adaptive uncertainty spaces. Non-adaptive elements of uncertainty spaces may be represented by a Markov chain (see Appendix A).

If only a few parameters of the uncertainty space are significantly influenced by the configuration vector and in turn significantly influence the configuration vector, it may prove useful to move these parameters from the uncertainty space to the configuration vector.

The initial state of the uncertainty space (at time 0) can either be fixed or stochastic. A stochastic approach may be beneficial if the initial conditions are not known accurately.

For example, the uncertainty space may include an element for the state of world conflict represented by a Markov chain. The world conflict state is assumed to take on a value from the following list of states:

1 = Peace
2 = Preparing for Conflict
3 = Regional Conflict
4 = Major War

A transition matrix can be developed based on historical data. For this example the authors created a data set by assigning one of the values from the above list to each year from 1900 to 2016. The probability of transitioning from one value to another from year to year is represented by a transition matrix P derived from the data set:

<i>P</i> =	[0.88	0	0.09	0.06]
	0.09	0.82	0.09	0.06
	0	0.12	0.79	0.13
	L0.03	0.06	0.03	0.75

Each element of the P matrix represents the probability of transitioning from the state corresponding to a column to the

state represented by a row. Note that since the probability of transitioning must be precisely 1.0, the sum of the elements of a column must equal 1.0.

Based on the original set of data, the probability of being in each of the four states is given by:

0.28
0.30
0.28
0.14

This vector of state probabilities can be used in a simulation to establish the initial state for the first year. The appropriate column of the transition matrix is then used successively to determine the probability to transition to one of the four states in the following years.

Table 2 provides five Markov Chains (labeled A through E) where the value for 2030 was generated by applying the overall probability of being in each of the states¹, and the successive year values were evaluated using the transition matrix. While each Markov Chain represents a different alternate future, each chain is statistically consistent with the original data set used to produce the transition matrix. Through this method, an arbitrary number of Markov chains may be generated of arbitrary length. For the short duration of the Markov Chains, the long term steady-state probabilities are not apparent. For this reason, one needs to employ multiple Markov Chains to adequately analyze the design vector, configuration vector, and uncertainty space.

Table 2: Example Markov Chains for State of World Conflict

Voor	Chain	Chain	Chain	Chain	Chain
real	А	В	С	D	E
2030	1	3	3	4	2
2031	1	3	3	4	2
2032	1	3	3	4	2
2033	1	3	3	2	2
2034	1	3	4	2	2
2035	1	3	4	2	2
2036	1	3	4	2	3
2037	2	3	4	3	3
2038	3	2	3	1	3
2039	3	2	3	2	3
2040	2	2	3	2	3

¹ See Appendix A for alternate methods for determining the initial value of the state. The long term steady state probability as derived from the transition matrix is given by:

0.27
0.32
0.27
0.14

COMPARING ALTERNATIVES

Each instance of a design vector is an alternative. The alternative is repeatedly exposed to the set of uncertainty space trajectories which results in multiple Markov chains of configuration vectors. A number of different methods (Monte Carlo Method for example) can be employed to generate the multiple Markov chains of configuration vectors. If a nonadaptive uncertainty space is employed, then multiple uncertainty space trajectories can be developed independently of the configuration vector (and alternative).

The configuration vectors for each design alternative are applied to each of the uncertainty space trajectories and evaluated for affordability (constrained or unconstrained) and operational relevance at each time increment. In any year, operational relevance for a specific configuration vector would normally be the result of warfare modeling of the capabilities represented by the configuration vector when placed in the conflict environment for that year as described in the uncertainty space. This performance could be characterized by one of four levels:

- Superior: The capability of the configuration is much greater than needed to perform its missions when an opponent (if any) has the capabilities described in the uncertainty space.

- Acceptable: The capability of the configuration is sufficient (but not much greater than needed) to perform its missions when an opponent (if any) has the capabilities described in the uncertainty space.

- Not Acceptable Constrained: The capability of the configuration is not sufficient to perform its missions when an opponent (if any) has the capabilities described in the uncertainty space. The technology exists for acceptable performance, but funding or schedule was insufficient to incorporate the technology into the configuration.

- Not Acceptable Unconstrained: The capability of the configuration is not sufficient to perform its missions when an opponent (if any) has the capabilities described in the uncertainty space. The technology does not exist for acceptable performance.

Within a design study, many uncertainty spaces would be developed. The number would depend on the ease of creating and evaluating the configuration vectors. Ideally, hundreds or thousands of configuration vectors for each alternative (design vector) would be developed and evaluated. The key is to base decisions on many possible uncertainty space trajectories instead of focusing on only a single possible future. One way of depicting these results is shown in Figure 4. For each time increment, a stacked column chart shows the fraction of configuration vectors that are evaluated in each of the different categories. In this example, alternative 3 has the highest probability of acceptable performance as compared to the other alternatives. Note that in any given year after the first year, the configurations for each alternative need not be identical. Each configuration would evolve based on how the modernization strategy reacts to each of the uncertainty spaces.



Figure 4: Alternative Comparison of Operational Relevance for a specific time

Comparing alternatives later in their service lives (Figure 5) can also provide valuable insight. Alternative 1 indicates a design vector that is not sufficiently flexible or adaptable; fiscal constraints often preclude incorporating the technology required for acceptable performance. Alternative 2 reflects a weak science and technology / research and development process that often is not capable of producing the technology needed for acceptable performance. Alternative 3 reflects a design vector which can usually adapt to the evolving uncertainty space in an acceptable manner. If a service life of 20 years or greater is desired, alternatives 1 and 2 are not likely to achieve the desired service life; they will likely be retired early due to unacceptable performance.





MODELING THE FRAMEWORK

Figure 6 illustrates one possible way of implementing the framework. A design vector development tool is used to create alternative design vectors for comparison. As stated earlier, this design vector consists of the initial ship configuration at delivery, initial tactics to employ the ship and the modernization process. Separately, an uncertainty space development tool creates a set of uncertainty spaces. Each uncertainty space specifies parameters such as a future adversary's capability in a warfare area, future technology breakthroughs, or the conflict environment. The configuration vector development tool applies the modernization process of

each design vector to each of the uncertainty space to develop a set of evolving configurations called the configuration vector: Each uncertainty space has a corresponding configuration vector for each alternative. The configuration operational relevance evaluation tool calculates how well the modernization process is able to respond to each of the uncertainty spaces. It produces the graphs similar to Figures 4 and 5.

The tools depicted in Figure 6 do not exist in a form that can be immediately used to implement the framework. A number of existing tools can be adapted to fulfill some of the required functionality, but additional work is needed to fully implement the framework.



Figure 6: Proposed Modeling Framework

CONCLUSIONS

Naval ship designers and force structure planners face high and rising costs in naval construction, and an unpredictably evolving future geopolitical and naval warfare environment. This paper introduces the framework for a new, quantitative approach to evaluating the benefit of modularity, flexibility, and adaptability features in the context of an uncertain environment and an adaptive modernization process.

The approach is grounded in the Real Options concept. Design vectors define the initial ship configurations, and configuration vectors capture changes to the designs over time, which are evaluated within sets of uncertainty space trajectories, to determine their economics and operational relevance through time. The results of the analysis are displayed in a set of graphs that portrays the effectiveness of different adaptive strategies to affordably adapt to its changing environment.

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APPENDIX A: INTRODUCTION TO MARKOV CHAINS

A Markov process, named after Andrey Markov, is a stochastic process where the system under study has multiple states, and the transition from the current state to the next state in a time increment is stochastically dependent on the current state, but not upon any previous (or future) states. For example, Figure 2 depicts a three-state process where the states are represented by the letters A, B, and C. The arrows represent the possible state transitions and the associated number is the conditional probability that given that the system is in the state at the base of the arrow, the transition will occur to the state at the end of the arrow during the following time increment. The sum of the probabilities of the arrows leaving a state adds up to 1.0; there is a 100 percent probability of transitioning, including transitions to the same state. For Figure 2, if the current state of the process is state A, then there is a 70 percent chance that the process will remain in state A, a 20 percent chance that the process will transition to state B, and a 10 percent chance that the process will transition to state C.



Figure 2: Example Markov Process

Figure 2 can also be represented by P, a transition matrix²:

$$P = \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.4 \end{bmatrix}$$

As a result of the constraint that the probability of transitioning from a state is 100 percent, the sum of the elements in each column of P is equal to 1.

If x_n is a stochastic vector with elements equal to the probability of system being in each of the three states at time n, then the probability of the system being in each of the three states x_{n+1} at time n+1 is given by:

$$x_{n+1} = Px_n$$

For example, if the system is currently in state A, then the current stochastic vector is

$$x_n = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

In the next time increment the probability of being in each state is given by:

$$x_{n+1} = \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.2 \\ 0.1 \end{bmatrix}$$

Similarly, the probability of being in each state at time increment n+2 is given by:

$$x_{n+2} = \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.2 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.62 \\ 0.23 \\ 0.15 \end{bmatrix}$$

 x_{n+2} can also be calculated by multiplying *P* by itself before multiplying it to x_1 .

$$\begin{aligned} x_{n+2} &= \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0.62 & 0.56 & 0.48 \\ 0.23 & 0.25 & 0.27 \\ 0.15 & 0.19 & 0.25 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.62 \\ 0.23 \\ 0.15 \end{bmatrix} \end{aligned}$$

Note that the stochastic vector at n+m can be determined by applying the *P* transition matrix *m* times...

$$x_{n+m} = P^m x_n$$

Hence if the current value of the state is known (x_n) this equation enables one to stochastically calculate the value at a desired starting year in the future (x_{n+m}) and thus provides a

² In a number of references, the transpose of this matrix is called the transition matrix. In this format, x_n is a row vector rather than a column vector depicted above: $x_{n+1} = x_n P$

method for determining the initial value of a Markov chain. Another method is to observe the system over some time period, calculate the probability of being in each state, and apply the resulting probabilities to determine the initial value.

If *m* becomes very large, P^m converges to the following matrix

$$\lim_{m \to \infty} P^m = \begin{bmatrix} 0.58 & 0.58 & 0.58 \\ 0.24 & 0.24 & 0.24 \\ 0.18 & 0.18 & 0.18 \end{bmatrix}$$

because the columns of this matrix are identical:

$$\lim_{m \to \infty} x_{n+m} = \begin{bmatrix} 0.58\\ 0.24\\ 0.18 \end{bmatrix}$$

Hence the long-term steady state probability of being in each state is independent of the original state and a function only of the transition matrix. This can be shown through an eigendecomposition of P to be generally true for the types of transition matrices normally encountered (see for example Ginstead and Snell 1997). One can use this long-term steady state probability as an alternate way to determine the initial value for a Markov chain.

REFERENCE

Grinstead, C. M., and J. L. Snell. 1997. *Introduction to Probability*, Providence, R.I.: American Mathematical Society.